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Screen-Reader Based Contextual Exploration of Mathematical Formulas in Brazilian Portuguese: Design, User Evaluation and Teaching Scenario in the Context of Numerical Analysis

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Abstract Although screen readers have made significant technological advancements, mathematics remains a challenging subject for people with visual disabilities. Due to its complex notations and abstract nature, mathematics presents difficulties in understanding through means other than visual. Consequently, reading mathematical content with screen readers poses challenges such as ambiguity, comprehension of long formulas, and identification of specific elements. Furthermore, even with reading difficulties, few screen readers support reading this type of content in Portuguese. This study presents an extension of a previous study which described the development and evaluation of an add-on for NVDA, which enables contextual exploration and navigation of mathematical formulas. The add-on, called Access8Math-NavMatBR, allows for internal exploration of formulas by providing contextual delineations of mathematical elements with support for the Brazilian variant of the Portuguese language. Based on the open-source Access8Math add-on, the new version was developed and evaluated in usability tests with six people with visual disabilities. Results showed that the new system improved understanding of formulas and provided better access to specific elements through formulas abstraction. The evaluation identified 52 issues, such as problems with commands and interaction approaches, verbalization by the screen reader, and platform structure. This extended version extends the analysis by presenting a teaching scenario in the context of numerical analysis and how the contextual exploration can be applied to aid in the understanding of complex elements. The paper presents design implications for systems for reading mathematical formulas in the Brazilian context and considerations for exploring patterns used by Brazilian users when reading and browsing mathematical formulas, dialoguing with the practical example presented.

Keywords: *Screen reader, Mathematical formulas, Visual Disabilities*

1 Introduction

Mathematics encompasses numerous notations, reading forms, and representative meanings for various fields of study. Therefore, understanding mathematical content goes beyond analyzing its individual elements, but also relies on a higher-level reading to identify the contextualization of each element and give it meaning within the studied scope. This neuro-visual process, despite being challenging, is automatically performed when reading a mathematical formula through vision. This is due to the ability to utilize the macro vision of the formula's content. However, for a visually impaired person, this process becomes more complicated.

Currently, there are a variety of screen readers and specific systems for visually impaired individuals in various fields, such as education. Among these, one can mention the NVDA[1] (NonVisual Desktop Access), JAWS[2] (Job Access With Speech), and ChromeVox[3] (the default reader for Chrome OS and available as an add-on for Google Chrome).

Screen readers perform reading through speech synthesis, which is transmitted linearly and sequentially, without the possibility of obtaining an overall view, as with visual means (Mejía et al., 2021). Due to the current fragility of Assistive Technology (AT) models and resources, users have less favorable conditions for obtaining an overall view of mathematical formulas, requiring memorization of the content with linear and sequential reading. To obtain an overall view effect, similar to that obtained by individuals without visual impairments, visually impaired individuals need to rely on keyboard navigation resources that anchor themselves in structural elements of digital content to navigate through the different parts of the content. This can be done on web pages with elements such as headers, lists, and others used in site navigation.

In order for web content to be usable by people with disabilities, it is important to follow standards such as the Web Content Accessibility Guidelines (Kirkpatrick et al., 2018). However, even though standardization helps with better navigability on web pages, there is still much to be studied regarding navigation strategies, as there are few works related to these techniques (Power et al., 2013; Watanabe, 2007, 2009). The number of studies is even lower in relation to the exploration of mathematical formulas.

In the current scenario, the literature presents three main ways of exploring mathematical formulas: linear exploration, tree exploration, and contextual or content abstraction exploration. However, despite the existence of different strategies for exploring mathematical formulas by screen reader users

[1]Available at http://www.nvaccess.com
[2]Available at https://www.freedomscientific.com/JAWS
[3]Available at http://www.chromevox.com/

in the literature, there is no knowledge about their usability for users in Brazil. The availability of resources for reading mathematical formulas in Brazilian Portuguese is very recent, and it is not yet supported by the main screen readers used in the country. Therefore, it is important to understand how visually impaired people with little exposure to navigation methods in mathematical formulas can interact, and what types of implications for the design of interaction resources with mathematical content read by screen readers are important for the blind population in Brazil.

A previous study from the authors of this paper (Guedes et al., 2022) presented the design and user evaluation of the extension of an existing add-on to read mathematical formulas using contextual exploration in screen readers. Despite presenting important implications for the design of the navigation and interaction strategies, the paper had limitations in the discussion of real-world use of the scenarios.

The present paper extends the study presented by Guedes et al. (2022), by providing further details about the qualitative analysis of the feedback provided by users with visual disabilities and detailing a real-world teaching scenario with examples from the course of Numerical Analysis.

The paper extends the report on the development and evaluation of a prototype add-on for screen readers that enables contextual exploration of mathematical formulas on the web for people with visual disabilities. The article presents implications for designing interactive navigation of mathematical content for people with visual disabilities, specifically in the context of users in Brazil who have limited exposure to screen readers for reading digital format formulas. It also presents a critical analysis from a professor of Numerical Analysis with examples of the use of the contextual exploration in the teaching of the Trapezoid rule for numerical integration. The analysis provides important insights into how the interaction approach can be used to aid in the understanding of complex concepts conveyed in mathematical formulas.

The remainder of this paper is organized as follows. Section 2 presents background concepts related to different strategies to explore mathematical content. Section 3 presents the methods for the design and usability evaluation of the add on implemented with contextual navigation. Section 4 presents the results of the usability evaluation. Section 5 presents the teaching scenario for numerical analysis. Section 6 presents discussions and limitations, and, finally, Section 7 presents conclusions and future work.

2 Mathematical content exploration strategies

The use of assistive technology (AT) by people with disabilities has significantly advanced. Some works have focused on creating tools for reading mathematical formulas (Stevens et al., 1997; Sorge et al., 2014; Soiffer, 2015; Doush and Pontelli, 2009; Soiffer, 2005, 2007; Frankel et al., 2014; Salamonczyk and Brzostek-Pawlowska, 2015; Cervone et al., 2016; Boonprakong et al., 2017; Maćkowski et al., 2018), while others aimed at creating tools that enable not only reading but also editing of formulas (Gaura, 2002; Edwards et al., 2006; Gruber et al., 2016; Bier and Sroczyński, 2015; Elkabani and Zantout, 2015; Rahman, 2005; Matoušek et al., 2011; Fajardo-Flores and Archambault, 2014; Ferreira and Freitas, 2005; Gulley et al., 2017). There are also tools that allow the generation of formulas in LaTeXwith accessibility features (Bier and Sroczyński, 2015; Ahmetovic et al., 2020). Some are capable of converting images of mathematical formulas into other more accessible formats such as LaTeXand MathML (Suzuki et al., 2004) or even used in e-books, such as EPUB3 (Brzostek-Pawłowska et al., 2019; Yamaguchi et al., 2014).

For the reading of mathematical formulas, in addition to screen readers (Edwards, 1989), braille lines have a long-standing history of use (Sutherland, 1972). There are three primary strategies for exploring mathematical formulas: linear, tree-based, and contextual or abstraction of complex content.

2.1 Linear Exploration

Linear exploration is the simplest strategy, in which the user reads the formula sequentially and linearly, without any structural treatment of the content. Concerning this strategy, Sorge et al. (2014) describe that it does not allow the user to have a greater ability to understand and explore the content due to its limitation in interaction. Figure 1 illustrates how linear reading occurs. In this example, a possible linear reading could be: "*the square root of three x minus one divided by two plus open parenthesis one plus x close parenthesis squared*". The linear model allows for limited interaction between the user and the formula, and for this reason, the reading occurs in a direct and uninterrupted manner.

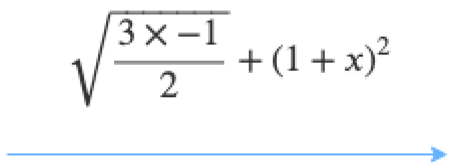

Figure 1. Representation of the linear navigation model

Edwards et al. (2006) developed the Lambda system with speech synthesis and Braille for linear exploration and reading. However, this method requires the user to memorize a larger amount of data, and it is difficult to go back or forward to parts of the equation or to jump to a specific section of the formula content. As a result, among users who used Lambda, there was no preference for reading content in speech-only mode, but there was unanimous preference for representation in Braille. Nonetheless, these resources are costly for many people, while screen readers are easier to obtain, including free options.

Junior (2021); Junior et al. (2020) proposed Math2Text, a web browser add-on that inserts textual descriptions of mathematical formulas as alternative text for formula images. The authors evaluated the perspective of including descriptions of mathematical formulas as alternative text, but did not provide details on the navigation and interaction that occurred during the study.

2.2 Tree exploration

In tree exploration, the mathematical content is treated as a tree structure where each element of the equation is a node, facilitating interactive navigation. However, a full understanding of the strategy is closely linked to the understanding of the tree data structure itself (Stevens et al., 1997; Sorge et al., 2014). Figure 2 exemplifies a possible tree representation of Formula 1, as well as its possible granularities. Navigating through granularity 1 would result in a linear reading of the equation; moving to granularity 2, the user would be able to jump between the square root, the $+$ sign, and $(1+x)^2$.

$$\sqrt{\frac{3x-1}{2}} + (1+x)^2 \tag{1}$$

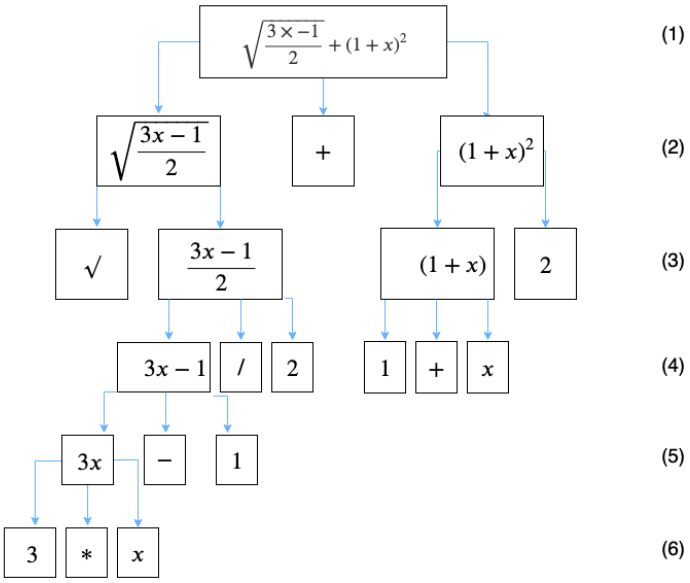

Figure 2. Representation of the tree-exploration approach

Raman (1994) was one of the first authors to create a reading and exploration system involving tree structures. The approach was incorporated into a text-to-speech (TTS) system - ASTER, which still influences research in the field today. JAWS, for example, uses tree exploration to read mathematical content. In this approach, the user has the same ability to explore as in tree structures (e.g. go to the next sibling element, go back to the previous sibling, go to the parent element, etc), as shown in Figure 2.

Several studies discuss the need for prior knowledge of the tree data structure for understanding content using tree exploration (Stevens et al., 1997; Gaura, 2002; Sorge et al., 2014). Sorge et al. (2014) highlight that tree exploration may be inadequate depending on the complexity of the expressions in the mathematical formula.

In order to analyze the efficiency of tree exploration strategies, da Paixão Silva et al. (2017) conducted a study on the main commercial screen readers (ChromeVox, JAWS, and a version of NVDA using linear reading) regarding the exploration of mathematical formulas. The research analyzed the execution of three tasks on a quadratic equation and their respective time efforts. Among the tasks were the verification of the existence of two identical real roots, determination of the concavity of the parabola, and analysis of the completeness of the equation. da Paixão Silva et al. (2017) used the GOMS (*Goals*, *Operators*, *Methods*, and *Selection rules*) task model and KLM (*Keystroke-Level Model*) to analyze the efficiency of tree exploration strategies. The authors estimated the effort required to complete three tasks related to a quadratic equation and measured the corresponding execution times using the commercial readers ChromeVox, JAWS, and a version of NVDA that uses linear reading. The results showed that completing the tasks using the different readers would take approximately 203.28 seconds for NVDA, 153.56 seconds for ChromeVox, and 132 seconds for JAWS.

Subsequently, da Paixão Silva et al. (2018) conducted a comparison with a software prototype based on ChromeVox. The prototype presented different proposals from the standard exploration of ChromeVox, including specific commands for mathematical equations. The study demonstrated that these features theoretically reduced exploration time. However, empirical data were not available.

2.3 Contextual exploration

Proposed by Stevens et al. (1997), the contextual exploration strategy is based on tree exploration. However, in contextual exploration, elements are not presented as tree nodes. In order for mathematical elements to be read and explored, the formula undergoes semantic grouping to separate each section of the formula into mathematical contexts. This is done so that navigation is performed on the mathematical groupings, such as fractions and roots, rather than on the tree nodes.

As illustrated in Figure 3, contextual exploration using semantic grouping enables the abstraction of all momentarily unnecessary information for reading, thereby reducing the amount of information conveyed. By doing so, the user is able to have an overview of the formula without the need for a complete reading. In addition to abstracting the formula, contextual exploration allows the user to navigate internally within each of the separated mathematical contexts and, thus, read its details.

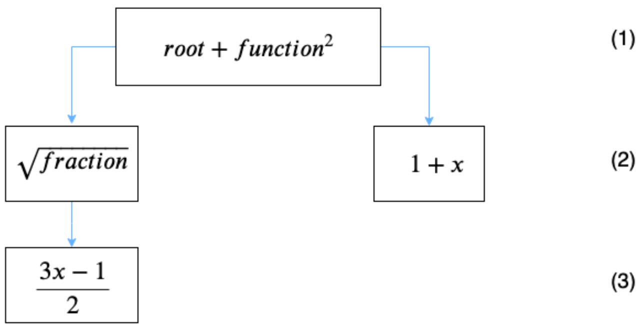

Figure 3. Representation of the contextual exploration model

In the contextual exploration strategy, the components of formulas are structurally separated into simple and complex terms (Stevens et al., 1997). Complex terms are sections with more than one term involved, such as a fraction, for example. This definition is important for defining the groupings and how to perform their reading.

In this strategy, reading is initially done linearly, where each simple term is read and complex terms are replaced by a representative meaning. Thus, when faced with Formula 2, for example, the content would be read as ”x is equal to a fraction”. The detailed reading of the ”fraction” content can be triggered by the user. In this case, the screen reader would be directed to the context of the ”fraction” and read the abstracted content. In addition to abstracting the content, the proposal aimed to make the user an active agent in the formula being read, improving the reader’s experience.

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \tag{2}$$

Different tools described in the literature implement the contextual exploration strategy, such as MathTalk (Stevens et al., 1997), MathPlayer (Soiffer, 2015), and MathJax (Cervone et al., 2016). In the literature, there are also works that present design considerations that address: the use of keyboard shortcuts as a means of aiding navigation (Stevens et al., 1997), granular visualization of formula components as a means of abstracting complexity (Fajardo-Flores and Archambault, 2014), interoperability of formula readers with various programs such as the Office package (Soiffer, 2015), presentation of expressions described in MathML language (Cervone et al., 2016), and development of frameworks to assist in the teaching of visually impaired individuals (Gulley et al., 2017). According to Gulley et al. (2017), the use of complexity abstraction in reading provides a reduction in memory overload. This information is relevant and reinforces the ideas presented by Raman (1994) and Fajardo-Flores and Archambault (2014).

None of the related works that use the contextual exploration strategy provide support for reading mathematical formulas considering Brazilian Portuguese. Despite innovative methodologies on exploring mathematical formulas, issues with ambiguity, handling long formulas, and generic treatment of formulas for navigation have not yet been solved.

3 Methods

This study was conducted based on previous research, which included interviews with teachers and individuals with visual impairments (de Lima et al., 2019), analytical studies with screen readers (da Paixão Silva et al., 2017; da Paixão Silva et al., 2018), and user testing with another screen reader (Abreu et al., 2019). In this study, a prototype Access8Math-NavMatBR was implemented with contextual exploration features using NVDA and evaluated with users. The protocol was approved by the Ethics Committee of the Federal University of Lavras, with protocol CAAE 65451917.8.1001.5148.

3.1 Prototype implementation

In this work, we present the development of an add-on for reading mathematical formulas in Brazilian Portuguese for the NVDA screen reader, based on the Access8Math add-on[4] developed by Tseng (2018). NVDA was chosen for the development of the prototype due to its widespread use in Brazil (Everis Brasil, 2018) and its support for reading in both web and desktop environments. Access8Math is an open-source add-on that allows developers to work on the code for its evolution. The version of the add-on reported in this paper was a branch of Access8Math, available at https://github.com/ManolloGuedes/Access8Math.

The prototype was implemented based on the contextual exploration and abstraction of complexity strategy proposed by Stevens et al. (1997), which is also used in more recent versions of MathPlayer (Soiffer, 2015). Thus, in addition to allowing user interaction with the formula, the reader is able to abstract complex information from the formula, providing only what is actually necessary at the moment of reading. To achieve this, the processing of the formula starts with defining what complex and simple elements are. In this work, we adopted that a complex element is a mathematical structure formed by two or more structural elements, while simple elements contain only one element.

Thus, in the reading dynamic, when facing a formula, the screen reader would have two options:

1. Read the element, if it is simple.
2. Announce the type of structure, if it is a complex structure, such as a fraction.

In relation to interaction with the formula, in the prototype, the same exploration patterns defined by the NVDA screen reader were implemented, with the addition of only one shortcut (defined with the keys CTRL + ALT + A) for activating and deactivating the complexity abstraction. The use of NVDA’s standard keys, and the reduction in the creation of new shortcuts were implemented to minimize the user’s learning and adaptation curve.

In order to implement the add-on, MathML (Miner et al., 2014) was used as it is a recommended XML markup language by W3C, aimed at describing mathematical notations with a structure that denotes their semantics. Furthermore, current screen readers that offer support for reading mathematical formulas are implemented based on this language, which allows working with well-defined mathematical structures. This is because the language is composed of specific tags that determine the type of described element. In the context of this work, the fact that MathML works with tags of structural definition provided means to define elements as being complex or simple.

Figure 4 illustrates the MathML encoding of Formula 2. The lines in Figure 4 have been numbered to facilitate analysis. Table 1 presents some lines exemplifying the characterization that will be made by the exploration prototype presented here based on the MathML structure of the formula in question. The elements used in the MathML code were:

1. **mrow**: group of sub-expressions;
2. **mi**: element designated as an identifier, such as function names, variables or symbolic constants;
3. **mo**: operator;
4. **mfrac**: container of a fraction;
5. **msqrt**: container of a square root.

[4] Available at: https://addons.nvda-project.org/addons/access8math.en.html

```
1 <mrow>
2   <mi>x</mi>
3   <mo>=</mo>
4   <mfrac>
5     <mrow>
6       <mrow>
7         <mo>-</mo>
8         <mi>b</mi>
9       </mrow>
10      <mo>&#xB1;<!--Sinal de mais ou menos--></mo>
11      <msqrt>
12        <mrow>
13          <msup>
14            <mi>b</mi>
15            <mn>2</mn>
16          </msup>
17          <mo>-</mo>
18          <mrow>
19            <mn>4</mn>
20            <mo>&#x2062;<!--Sinal invisível de multiplicação--></mo>
21            <mi>a</mi>
22            <mo>&#x2062;<!--Sinal invisível de multiplicação--></mo>
23            <mi>c</mi>
24          </mrow>
25        </mrow>
26      </msqrt>
27    </mrow>
28    <mrow>
29      <mn>2</mn>
30      <mo>&#x2062;<!--Sinal invisível de multiplicação--></mo>
31      <mi>a</mi>
32    </mrow>
33  </mfrac>
34 </mrow>
```

Figure 4. Source code of the Bhaskara formula using MathML

Table 1. Classification of mathematical exploration of complexity of the elements shown in Figure 4

Line	Element class	Justification
2	Simple	Atomic element x
3	Simple	Atomic element =
4	Complex	Group of elements nested under <mfrac>
7	Simple	Atomic element -
8	Simple	Atomic element b
10	Simple	Atomic element ±
11	Complex	Group of elements nested under <msqrt>
13	Complex	Group of elements nested under <msup>
14	Simple	Atomic element b

3.2 Usability evaluation of the Access8Math-NavMatBR with Contextual Exploration

Tests were carried out on the Access8Math-NavMatBR software system in order to verify usability issues. The protocol used in the tests was the think-aloud.

3.2.1 Participants

Six participants of different ages, educational backgrounds, and types of disability were selected to test the Access8Math-NavMatBR software system. Due to the limitations imposed by the new coronavirus pandemic, all tests were conducted remotely. Therefore, participants with experience with screen readers were selected. Table 2 presents the demographic data of the test participants.

The researcher who conducted the usability evaluations sent the informed consent in an accessible format beforehand, and explained any points of doubts regarding the research during the call. Participants were clarified that they could interrupt their participation at any point.

The recruitment of participants was done through a post in a Facebook social network group dedicated to people with visual impairments. The phone number of one of the authors of this paper was made available to schedule and clarify any doubts about the tests with the Access8Math-NavMatBR software add-on. All tests were recorded in order to enable the analysis of behavioral aspects that indicate comfort or discomfort regarding the tested software.

3.2.2 Tasks

In the work presented by Abreu et al. (2019), the authors conducted tests by providing four mathematical formulas to be read by the evaluated screen reader, and subsequently presented the same formulas to the participants in a tactile version, in order to verify the compatibility of the participant's perception between the two forms. The initial idea of this project was to use the same approach in order to make a better comparison of results. However, due to the isolation resulting from the Covid-19 pandemic, it was not possible to conduct the tests in person.

The tasks applied to the participants followed the same approach proposed by Abreu et al. (2019). However, the verification of perceptual compatibility through tactile reading was replaced by a questionnaire applied to each of the formulas to verify the accuracy of participants' understanding. Following is a list of the 4 formulas used in the tests. The formulas were chosen to cover different levels of abstraction and nesting of mathematical content. The formulas cover elements in simple parentheses, combinations of parentheses, powers, fractions, and elements within fractions with parentheses. These elements offer different levels of challenges for exploring mathematical content.

- Formula 1: $(3 - \frac{1}{2})$
- Formula 2: $-2^3 \times (40 \div \frac{18}{\sqrt{36}}) \times (3 - \frac{1}{2})$
- Formula 3: $\frac{32+\sqrt[3]{\frac{128}{2}}}{8+\sqrt{4}}\pi$
- Formula 4: $\frac{-4^6 \times (18 \div \frac{12}{\sqrt{25}}) \times (9 - \frac{1}{3})}{8^5 \times (\sqrt[4]{81} \div 3)}$

After reading and exploring the formulas, the tasks involved questions such as "What is the numerator of the fraction?" (open-ended, Formula 1), "Is there a square root in the formula?" (Yes, No, I don't know - Formula 2), "What is the degree of the root in the denominator?" (open-ended, Formula 3), and "What is the content of the first fraction?" (Formula 4).

3.2.3 Tests Conduction

The participants of the tests received an email containing:

- Informed Consent Form: the user was requested to read and respond to the email if they agreed to participate in the tests.
- Calibration file: an HTML file containing 2 mathematical formulas used to train the user on how to use the addon.
- Test file: an HTML file containing the mathematical formulas described in Section 3.2.2.
- Addon installation file.

Table 2. Demographic data of participants in the test with the Access8Math-NavMatBR software addon

Code	Age	Sex	Disability	Primary Screen Reader	Experience with Screen Reader	Education
D1	33	M	Congenital low vision	NVDA	11 years	Postgraduate degree
D2	27	M	Acquired total blindness	NVDA and VoiceOver	11 years	Postgraduate degree
D3	42	M	Acquired low vision	NVDA, VoiceOver and Talkback	12 years	Completed undergraduate degree
D4	32	M	Acquired total blindness	NVDA and VoiceOver	15 years	Undergraduate degree in progress
D5	32	M	Acquired total blindness	NVDA, VoiceOver and Talkback	9 years	Undergraduate degree in progress
D6	27	F	Acquired total blindness	NVDA	15 years	Completed undergraduate degree

All tests were conducted virtually through calls using Google Meet or WhatsApp, and audio recordings were made with the participants' consent.

At the time of the test, after installing the add-on, users were invited to open the calibration file to receive instructions on how to use the tool. The instructions included an explanation of the operation of the complexity abstraction exploration, as well as how to use shortcut keys. At this point, participants were asked to read the formulas and explore them as they wished for as long as they needed. The tests were only started after this first contact with the add-on. After the calibration stage, users were asked to open the test file and start reading each of the four formulas, followed by their respective questions.

Using recordings of the tests, we conducted an analysis of participants' reports regarding the use of the Acces8Math-NavMatBR software was conducted. From these reports and the researcher's observation of participants' behavior, 52 problem reports were recorded. The analysis of the reports was conducted using elements of the thematic analysis methodology Braun and Clarke (2006) in order to identify emerging themes. The process of thematic analysis involved a round of open coding conducted by the principal researcher of this work. After this stage, the supervising researcher reviewed the established themes, and a discussion was held regarding points of disagreement. The 52 problem reports were grouped into the following themes: Interaction with Formulas (22 instances), Formula Narration (27 instances) and Platform (3 instances).

The analysis of the reports was conducted using elements of the thematic analysis methodology Braun and Clarke (2006) in order to identify emerging themes. Within their respective themes, each problem record was further categorized following the same process of open coding, conferencing, and discussion, resulting in 26 different categories (12 categories for the Interaction with Formulas theme; 13 categories for the Formula Narration theme; and 1 category for the Platform theme).

The theme "Interaction with Formulas" encompasses the manipulation of the add-on and the mathematical formula by the user. This theme includes issues such as difficulty reaching the inside of a square root or enabling the abstraction mode, for example. The theme "Narration of Formulas" encompasses points related to the reading mode offered by the add-on. Examples of this include issues such as the speed of reading and misunderstanding of the words used to describe a certain element.

The "Platform" theme encompasses suggestions for making the add-on available for desktop environments and software, such as Microsoft Office Word.

The themes that emerged resulted from the mapping between the categories assigned in the open analysis of the tests. 25 categories were identified and discussed in Section 4.

4 Usability Evaluation Results

This section presents a description of the results encountered during the usability evaluation, with an analysis of each group of reports according to their themes.

4.1 Interaction with Formulas

A total of 22 problems were found regarding Interaction with Formulas, grouped into 12 categories, listed in Table 3.

The following are descriptions of each of the categories of problems observed in the tests.

4.1.1 Difficulty in triggering the expansion model in reduced formula

When reading the first formula with the abstraction mode activated, participant D1 became confused about the reading. When asked what the numerator of the fraction present in Formula 1 was, the participant said they could not identify the content of the fraction. As this was a problem that could completely impact the test's completion, an intervention was made to remind the participant that the content of the fraction would be presented through exploring the formula by entering into interactive mode.

Table 3. List of categories of problems related to Interaction with Formulas by occurrence and severity

Category	Instances	Severity
Difficulty in actvating the expansion model in a reduced formula	1	Severe
Difficulty in understanding that a complex element can have other complex elements inside	4	Severe
Unclear functioning of shortcut keys for exploration	3	Severe
Difficulty in understanding the abstraction of the formula into more general components	2	Simple
Difficulty in understanding how to increase or decrease the level of abstraction (zoom)	1	Simple
Difficulty in identifying the attribute of a root element	1	Simple
Difficulty in identifying a specific element in the detailed description of an element inside a fraction	1	Simple
Difficulty in exiting the reading of the expansion of a section of the formula	1	Simple
System does not allow customization of keys for contextual exploration	1	Simple
Did not understand that it was possible to explore the content of the abstract 'fraction" element	1	Simple
Did not understand that it was possible to explore the content of the abstract "root" element	5	Simple
Difficulty in understanding what the current level of abstraction (zoom) is	1	Cosmetic

4.1.2 Difficulty in understanding that a complex element can contain other complex elements

This category was extracted from four occurrences involving participants D1, D5, and D6. When reading the mathematical elements and encountering the fractions present in formulas 3 and 4, the participants were unable to understand that it was possible to have new mathematical expressions and complex elements within another complex element. Therefore, at times, the internal exploration to find sub-elements was not carried out.

4.1.3 Unclear functioning of shortcut keys for exploration

Three considerations were identified involving problems in understanding the shortcut keys for exploration, all obtained from participant D1's test. The first consideration was regarding Formula 3, in which, when exploring the fraction present in the formula, the participant thought that pressing the down arrow key would make a jump from the denominator to the numerator. In Formula 4, in order to read the elements inside the fraction, the participant tried to use the left and right arrow keys to navigate between the elements of the fraction without first expanding it to access its content. Still, in Formula 4, the participant tried to zoom in on the formula and expand the fraction using the up arrow key.

4.1.4 Difficulty understanding the abstraction of the formula into more general components

After entering into the interaction mode, participant D1 was confused about how to perform a detailed reading of the fraction in Formula 1. Below is a quote from participant D1 regarding the problem:

> "...when you start reading, it's a bit confusing...it [the screen reader] says 'three minus fraction', and then I thought: where is the denominator of the fraction?"

The difficulty in understanding the abstraction of the formula into more general components also occurred with participant D2, where Formula 3 was read multiple times until internal exploration of the formula was performed to search for details of the elements.

4.1.5 Difficulty understanding how to increase or decrease the level of abstraction (zoom)

During the reading of Formula 3, participant D1 tried to navigate between the elements that composed the denominator of the formula using the down arrow. The correct approach would be to use the left and right arrows, since the up and down arrows are used for entering and exiting complex element details. Below is a quote from the participant regarding the problem:

> "when you zoom in on the denominator, it [the screen reader] reads to me 'eight', and then says that it won't move because it can't read the square root that's there... I'm pressing the down arrow."

4.1.6 Difficulty identifying attribute of root element

To identify the attributes of the root, such as content and degree, it was expected that the participants would expand the root and enter its details. However, user D1 did not perform this process.

4.1.7 Difficulty identifying specific element in detailed description of element within a fraction

To identify the details of a complex element within another complex element, it was necessary to expand the desired element. However, when reading Formula 3, participant D1 did not navigate internally in the numerator elements to identify the content of the root present at that level.

4.1.8 Difficulty exiting the formula section expansion reading

In NVDA, users usually use the insert + space key combination to enable and disable edit mode. When entering interaction mode in Formula 1, participant D1 questioned whether it would be possible to exit using these keys.

4.1.9 System does not allow customizing keys for contextual exploration

For the use of the abstraction mode implemented in this project, the shortcut key combination CTRL+ALT+A was established for enabling and disabling the mode. However, these keys may override the definition of some other functionality already in use in NVDA.

4.1.10 User did not understand that they could explore the content of the abstract element "fraction"

While reading Formula 3, participant D1 encountered a fraction and was asked if there was any exponent in the formula. The participant stated that there wasn't, however, they did not expand the fraction to identify whether or not its content contained the requested information.

4.1.11 User did not understand that they could explore the content of the abstract element "root"

When reading the complex element root, participants D1, D3, and D5 had trouble identifying its content, as they did not remember that it was possible to explore the formula by navigating to the interior of these elements.

4.1.12 Difficulty understanding the current level of abstraction (zoom)

At the end of the test, participant D2 suggested that information be included regarding the current level of internal exploration of the formula. According to the participant, this information would be useful in assisting in their location within the formula.

> 'I realized that in the first test, I felt the need for 'steps' information, in the last ones I think I was a little more intuitive in navigating, ... still, I think it may be necessary [information about the levels], I think this will become a reference point."

4.2 Formula Narration

27 observations were identified regarding the narration, grouped into 13 different categories. Table 4 presents the categories and their respective occurrences.

The main categories of problems regarding narration observed in the tests are described as follows.

4.2.1 Difficulty in understanding narration of root index

Due to the fact that the index of the root can take any value in the set of natural numbers, its description was defined in a generic way for the wide variety of values that the index could assume. Therefore, it was decided to use "root to X" as narration, where X is the degree of the root. However, due to the difference between the pattern used by the prototype software and that used by participants, D2, D3, and D5 had trouble reading the root index.

Table 4. Categories of Formula Narration Considerations by Number of Occurrences

Category	Instances	Severity
Difficulty in understanding narration of root index	5	Severe
Reading of a list of enumerated elements can confuse users	2	Severe
Reading the numerator and denominator of a fraction without pause makes understanding difficult	1	Severe
Narration of a root is confusing for the user	1	Severe
Narration of one element at a time leads the user to believe that there are no more elements	3	Severe
Difficulty in understanding "negative" as "minus sign"	1	Simple
Difficulty in understanding the structure of a fraction by reading	2	Simple
Difficulty in understanding the narration of exponentiation due to different patterns used by teachers	1	Simple
Inconsistency in reading and announcing entry into the interior of elements	1	Simple
Difficulty in distinguishing what is division and what is fraction	4	Simple
Difficult narration to understand	1	Simple
Standard narration is too fast	1	Simple

4.2.2 Reading a list of elements with enumeration can confuse users

When entering the interaction mode and exploring the formula's elements using the directional arrows, the screen reader enumerates each of the elements being read to assist the user's location during reading. Thus, assuming the first element of the formula is 5, the screen reader would narrate: "item 1...5". However, participants D4 and D5 pointed out that this enumeration, being done using a number, can generate confusion about what is an element of the formula. Participant D2, on the other hand, liked the enumeration.

4.2.3 Reading the numerator and denominator of a fraction without a pause between them makes understanding difficult

According to participant D2, the pause between the narration of the numerator and denominator of a fraction is not satisfactory.

4.2.4 Narration of the root is confusing for the user

In reading formula 3, Participant D6 understood that the content of the root of the numerator was the Greek letter Pi, but this symbol was located outside the root. However, it was observed that the participant did not expand the root to verify its content.

4.2.5 Narration of one element at a time leads user to believe there are no more elements

At times, users D4 and D5 did not read the complete complex elements, assuming that there was only one element within the complex element, they only read the first term.

4.2.6 Difficulty in understanding "negative" as "minus sign"

Regarding negative numbers, the narration of the elements was made in the form of "negative X", where X is the negated element. This way of reading generated doubts in user D6, but was a point of praise for user D2.

4.2.7 Difficulty in understanding fraction structure through reading

When reading formula 4, participant D5 had difficulty identifying the first fraction within the numerator. The difficulty arose because the narration was done in the following way: "fraction with numerator...divided by fraction...". The fact that a fraction was read inside the outer fraction made the participant imagine that they had already exited the outer fraction and become lost in the formula.

Additionally, participant D4 questioned the fact that simple fractions did not present the narration: "numerator...denominator...", but narrated directly: "1 over 2", for example. According to him, maintaining the standard for these cases can facilitate comprehension.

4.2.8 Difficulty in understanding exponent reading in exponentiation

On four occasions, participant D5 had trouble identifying the exponent in the formula. However, the participant stated that they did not have knowledge of what an exponent is, and that was the reason they could not identify this type of element. This problem was not identified in the tests of the other participants.

4.2.9 Difficulty in understanding narration due to different patterns used by teachers

Participant D2 stated that due to the lack of standardization in the way mathematics is narrated - even among teachers - this would be a problem that users would possibly encounter.

4.2.10 Inconsistency in reading and announcing entry into the interior of elements

It was adopted in the add-on implementation that, when faced with a root containing only one element inside, upon expanding the root, the narration would be "root of X", where X is the content of the root. On the other hand, when reading roots that have several elements inside, upon expanding the root, the narration would start with the information "interior of the root". Participant D4 questioned the lack of standardization in the reading.

4.2.11 Difficulty in distinguishing between division and fraction

It was agreed that the narration of fractions, in an abstract way, would be done through the expression "fraction". However, some of the formulas used had divisions that did not qualify as fractions. The division operator was agreed to be narrated as "divided by". Participants D2, D5 and D6 had doubts about the reading at points where there were the division symbol, believing that the composition of the elements was a fraction.

4.2.12 Formula narration was difficult to understand

When reading Formula 4, participant D6 stated that he was confused by the formula, as it had several elements and several levels of navigation.

4.2.13 Standard narration is too fast

When expanding a complex element, a linear and sequential reading of its elements is performed, without user interaction. It was identified that participant D5 started to use the character by character reading, available natively in NVDA, to read the content slowly.

4.3 Platform

Three observations were obtained regarding the target platform of the solution proposed in this work. Participants D3 and D4 questioned whether, in addition to enabling the reading of mathematical formulas on the web, it would be possible to create add-ons capable of reading text editing tools such as Microsoft Office Word.

Participant D3 also questioned whether any type of formula available on the web could be read using the Access8Math-NavMatBR software. At this point, it was clarified that the add-on only supports formulas structured with the MathML markup language. The participant suggested a movement to disseminate the correct way of creating content using MathML.

4.4 General Considerations

Eight general considerations were obtained regarding the use of the Access8Math-NavMatBR software add-on, with two suggestions and six positive points regarding the proposed approach and the use of the add-on.

Participant D2 suggested that, along with the installable add-on, support material be made available on how each mathematical element is read and how the exploration strategy used is carried out:

> "It would be very cool [to create the support material] ... it becomes even a pedagogical part of the add-on. Instead of just developing and leaving the user to fully understand [how the exploration and reading is done], or that they always need guidance or someone alongside [when using the add-on]."

Participant D5 suggested that the description of mathematical symbols provided natively by NVDA be used. Participant D1 related the implemented contextual exploration method to the exploration method commonly used by visually impaired people in Excel tables. Having to find the denominator of a fraction before being able to explore its content is similar to navigating matrices in Excel. To read the content of a column, it is common to navigate to its header to find out about the content of the column being read.

In addition to their feedback, the participants expressed satisfaction in using the add-on to perform readings, for example:

> "... from this test today, I would be able to read all the equations, all the formulas I wanted, I would have the ability to [read] ..." *Participant D2*

> "... it's an incredible novelty [the add-on], a system that has a very interesting proposal because it is our need to do digital reading of mathematical formulas, and the [screen reader] still does not interpret this ..." *Participant D4*

> "... being able to expand the elements, zoom in, it's very interesting, especially because you managed to make linear reading possible ..." *Participant D4*

> "... I found the system fantastic, we see a great need in terms of mathematics for use with a screen reader, and the proposal [of the add-on] is very good ..." *Participant D5*

> "... it was easy [to read] ... breaking down each part of the formula, I thought it was good [to perform the reading] ..." *Participant D6*

In addition to these statements, according to participant D6, the way the reading was performed reduced memory overload and allowed for the habituation of the reading method. This fact was evidenced in the tests. Several times, when asked about the content of a formula segment, participants did not need to go back to the segment to remember its content.

Furthermore, during the use of the add-on, it was observed that participants began to feel more comfortable using the system. Regarding this, user D2 stated:

> "... the user starts appropriating the language, listening, processing, ... even though I have some experience with math, it's another way of reading. So there is also the process of the user adapting ... the user gets used to [using the system]"

During the tests, a very important report was made by participant D2, in which he stated that he was unfairly disadvantaged in a contest for a public job position due to the problem of ambiguity in reading mathematical formulas. Here is the transcription of that part of the dialogue:

> "In 2019, a contest was opened for a Federal University (job position), for a braille text reviewer... it was precisely for the adaptation of material into braille... among the required knowledge, there was knowledge of adaptation of mathematics, chemistry and Portuguese... it was a subject that I mastered... when it came to the test, I passed first in the theoretical test, and in the practical test, there was a situation where we had to adapt an equation that was on the computer screen... and the person had to write in braille linearly... for those who cannot see, there was a reader and also on the computer, the formula adapted in digital format for reading in NVDA... in these mathematics formulas, I had many ambiguities, various ways of interpretation that I couldn't figure out what was in what, where a fraction ended and where it didn't, if a sum was in the numerator or if it was already outside the fraction... I had several situations where I had to ask the reader... I couldn't make this distinction, there was no grouping or anything that informed me... and the person always told me that all they could do was read... I couldn't interpret the fraction that was described on the screen or by the person reading... and because of a detail, I failed this test... the test was impossible to understand."

After that, he made the following statement:

> "... it is because of this type of situation that I see the value and potential of such a well-done work as this one, reading mathematical equations, you can define someone's life. Accessibility is not simply saying that the blind person was able to read. Accessibility is ensuring that someone passes a contest... it is ensuring that the person uses what they knew, what they studied, what they acquired in life to sometimes pass an opportunity that was their chance. So, accessibility is no joke."

The statements made by participant D2 demonstrate the need and importance of having a system that is able to mitigate problems such as ambiguity, and the size of the impact generated by this type of problem.

In addition to the statement of participant D2, another important report was made by participant D3. According to what was reported, the participant has a degree in mathematics and was working as a mathematics teacher until he lost his vision, after which he chose to abandon his career as a mathematics teacher.

Regarding the use of NVDA's standard keys, all participants responded that it makes usage easier. Regarding the insertion of the new shortcut key for enabling/disabling contextual exploration, participants were asked if the keys were difficult to memorize. All responded negatively, stating that the keys were easy to memorize.

Two more questions were asked regarding the understanding of the formulas, with a 5-point Likert scale. In relation to the use of abstract navigation to reduce ambiguity in reading the formula, the median of the responses was 4.5. Regarding the improvement in formula comprehension, the median was 5.

4.5 Suggestions for improvements

From the tests, eight problems were identified as serious, with five related to narration and three related to interaction. The following are suggestions for possible improvements to be developed in the future of the project.

4.5.1 Difficulty activating the expansion model in reduced formula

Participants had difficulty remembering that it was possible to explore formulas internally and how to do this exploration. This type of problem could improve with more frequent use of the system, adjustments to commands and help resources.

Other problem categories include:

- difficulty understanding that a complex element may have other complex elements within it and
- the operation of shortcuts for exploration is not clear.

4.5.2 Difficulty in understanding narration of root index

The problems involving the narration of the root index were due to the words used to describe this element. Currently, a parallel study is being conducted by a student in the Mathematics Education course to identify the best ways to narrate specific mathematical elements. As pointed out by participant D2, there is no standard way of narrating each mathematical element.

4.5.3 Reading a list of enumerated elements can confuse users

Two participants pointed out that the enumeration of terms in a formula can hinder the understanding of mathematical content. One of the participants stated that enumeration helped him locate the internal parts of the formula and suggested that this enumeration occur not only on terms but also on levels of internal exploration of the formula.

One possible solution would be to give the user the option to enable or disable enumeration and to provide new forms of enumeration using letters or ordinals.

4.5.4 Reading the numerator and denominator of a fraction without pause between them makes understanding difficult

As a solution to this problem, different settings for reading speed and pause between term narration could be tested or allowing the user to have control over them.

4.5.5 Narration of one element at a time leads the user to believe there are no more elements

Both problems occurred due to the lack of visibility of what exists inside complex elements. As a solution to these problems, the narration of these elements could be modified to inform not only the type but also the quantity of elements that exist inside them. Thus, when faced with the square root of the Bhaskara formula ($\sqrt{b^2 - 4ac}$), it would be narrated as "square root with 4 internal terms." This solution would enable us to present the user with two versions of narration, a short one containing the current narration, and a detailed one containing the number of terms inside it.

The following section describes an analysis of a teaching scenario in the context of a topic in a Numerical Analysis course, present in many degrees in Engineering, Physics and Technology degrees. The section extends the discussions raised in the previous study on which this paper builds upon (Guedes et al., 2022), bringing pratical examples of the use of the contextual exploration approach in a concrete example.

5 Teaching Scenario in the Context of Numerical Analysis

Numerical Analysis is a mandatory discipline in a range of Science, Technology, Engineering and Mathematical degrees. This discipline involves a number of methods to solve mathematical problems using techniques that can be implemented in computer algorithms.

Many such methods have complex formulas with important components whose semantics are fundamental to the comprehension of the concepts. One such area in Numerical Analysis is numerical integration.

Practical applications of numerical integration include calculating areas, analyzing flows, determining averages over continuous ranges, calculating centers of gravity for irregular objects, determining total quantities of physical variables, and finding effective currents, all of which are fundamental problems in engineering practice. The mathematical representation of the area under a curve f(x) is symbolically denoted by the integral, according to the equation 3:

$$\int_a^b f(x)\,dx \tag{3}$$

The following teaching scenario describes the semantics of a specific set of methods for numerical integration and the cognitive path students need to go through to understand how they work. The example shows a number of occasions in which complex formulas may become complex to understand for students with visual disabilities. In such case cases, a full linear reading of the formulas may make the understanding of specific features of the formulas and the methods difficult to understand.

The description shows situations in which the contextual exploration of formulas by screen readers may provide features to identify crucial components in formulas that are essential to the understanding of the methods.

Regarding mathematical interpretation, two things are fundamental in reading 5: identifying the integral as being defined in an interval, with inferior and superior limits a and b, respectively, and determining the algebraic definition of the function $f(x)$. This concept is essential for the study of integrals in general, not just from a numerical point of view. Such are examples of complex elements that can be identified in overall readings of the formula before delving into the full linear reading of their components.

One of the methodologies used in numerical integration is the so-called closed Newton-Cotes formulas. The formulas are called closed because the values of a and b (the endpoints of the integration interval) are known. The purpose of numerical integration is to replace a mathematically complex function with a simpler approximating function that is easier to integrate. Some examples of closed formulas are Trapezoid rule, Simpson's rule, Simpson's 3/8 rule, Boole's rule, Weddle's rule and Milne's rule. Each of these formulas uses a specific number of integration points within the integration interval and has an associated order that determines the accuracy of the approximation.

The formulations for executing the Trapezoid rule without repetition are more straightforward from the point of view of reading the generic formulation. The result of integrating a first-degree interpolating polynomial is equivalent to the formula for the area of a single trapezoid, as can be seen in Equation 4:

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + f(b)\right] \tag{4}$$

As usual in mathematics, the formula is read from left to right. Considering the context of numerical analysis teaching, to better understand the concept, the formula reading can be divided into two parts that we will call "regions" here. The first region is outside the square brackets, which relates the calculation of the half of the interval h, whose value is determined according to the extremes of the integration interval ($h = b - a$), which represents the geometric height of the trapezoid. The second region is the calculation inside the square brackets, which represents the sum of the edges (bases) of the trapezoid, which we usually call "major base" and "minor base". In 4, these bases are represented by $f(a)$ and $f(b)$. That is, the values of the bases are the function applied to each of the limits of integration.

The difficulty in reading and applying the formula in the numerical solution of the integration problem by the trapezoid method will also depend on the algebraic complexity of the function $f(x)$.

Let us analyze two situations considering two $f(x)$ formulas with different levels of algebraic complexity:

1. Integrate the function

$$f(x) = x^2 + 2x - 10 \tag{5}$$

   in the range from 1 to 5, using the Trapezoid Rule
2. Integrate the function

$$f(x) = \frac{5x^2}{\sqrt[3]{x^3 + 4}} \tag{6}$$

   in the range from 1 to 5 using the Trapezoid Rule

Considering problem 1, the integral to be solved is:

$$\int_a^b x^2 + 2x - 10\,dx \tag{7}$$

According to formulation 4, to apply the Trapezoid rule, the following steps must be followed:

- Identify the integration interval for calculating the h spacing. In this case, $h = b - a = 4$.
- Compute the first region, i.e. half of h.
- Identify integration notation and range endpoints.
- Identify the algebraic form of the function $f(x)$.
- Apply the algebraic form at the extremes of the interval, that is, calculate $f(a)$ and $f(b)$.
- Calculate the sum of $f(a)$ and $f(b)$, calculating the second region
- Multiply the result of the first region by the second region.

In this case, the continuous in line reading of the formulation would be "integral from 1 to 5 of x squared plus two times $x - 10$ of x". For the sighted person, reading is supported by the graphic visualization of the mathematical symbology of the formula, which properly indicates the algebraic limits, such as only x being squared. In the case of a person with visual impairment, continuous line reading leaves doubts as to whether the exponentiation only applies to x, or if it also includes the addition and subtraction involved. A visually impaired person navigating the mentioned regions would have more clarity of the steps.

Appropriate definitions of these regions can be inspired by the mathematical context of solving equations according to the priorities of each mathematical operation. Considering Problem 1, the ideal reader would be for the screen reader to identify five regions or blocks, identified as follow:

- Region 1: Integration limits a e b identification, aiming the h evaluation;
- Region 2: Identification of the portions of addition and subtraction operations
- Region 3: First addend reading of the base of the exponent, with indication of its beginning and end, followed by expoent reading;
- Region 4: Second addend reading, identifing the operation involved.
- Region 5: Third addend reading, which allows the final operation between the three parcels.

The separation in regions or blocks reading strategy could be compared to manipulable resources used in teaching to students with visual disabilities. In this case, the tactile interaction of the material promotes greater student engagement, increasing their autonomy and providing a spatial understanding of the problem solution steps. The identification of the region can help the visually impaired student to associate the mathematical formulation with blocks, promoting this spatial problem understanding. This makes this student have a more comprehensive mental representation of the operations involved, in addition to reducing the occurrence of calculation errors.

In Problem 1, the tool in this paper could help with identification of the different steps by identifying the elements in an integral (upper and lower) and the mathematical operations. If the addends are simple, the system would read them as a single block. However, the tool offers resources to teachers, so they can design their MathML in a way students could break down the addends. A possible alternative to this is to add parenthesis to include a structural element.

It is important to consider that the higher complexity of the algebraic formulation to be read, the greater the need to identify well-delimited regions, as can be seen in Problem 2. In this case, the execution is quite similar, but the complexity of the problem is more significant due to the algebraic complexity of $f(x)$. Thus, in addition to considering the two calculation regions of the method formulation already mentioned, it is also necessary to think about the regions inherent to the algebraic composition of 2. It would be necessary to identify the regions involving the numerator and denominator of the fraction, exponentiation, roots, indices, radicals, and radicands for calculation. Sighted students make these identifications by observing the graphical representation of the formula. In the case of students with visual impairment, using a screen reader that is unable to navigate through the formulation and identify these regions can seriously compromise the execution of this type of formulation.

Thus, the ideal reader for Problem 2 would identify five regions, as follow:

- Region 1: Integration limits a e b identification, aiming the h evaluation;
- Region 2: numerator identification
- Region 3: denominator identification
- Region 4: operations identification, including multiplication factors, sums installments, exponentials operations, etc.
- Region 5: Root indentification, considering the operations involved in radical e radicand.

Note that the number of regions or blocks identified is directly proportional to the algebraic complexity of the function being analyzed. Thus, the navigation of screen readers through the regions of the formula are of fundamental importance for their spatial identification. This correct spatial identification helps the visually impaired person to perform the operations in a more accurate and detailed way, improving the understanding of the formulation.

In the case of Problem 2, the identification of lower and upper limits can also be done by using structural elements. The use a fraction also enables the breaking down of denominator and numerators, as with the identification of the different parts of a root.

It is essential to point out that this analysis is limited to calculating the trapezoidal rule without repetitions. When it is necessary to increase the reliability of the approximation, the use of closed Newton-Cotes formulas in their repeated form is indicated, for example:

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\sum_{i=1}^{n+1} f(x_i)\right] \tag{8}$$

where $h = \frac{b-a}{n}$, and n is the repetitions Trapezoidal rule number.

This further increases the need to analyze the execution of formulations across reading regions. In the case of numerical analysis, identifying these regions can also improve the translation of the mathematical formulation into pseudocodes that can be computationally implemented, which is common in teaching these topics. The Trapezoidal rule, and also the other closed Newton-Cotes formulas, may not be executed correctly if the algebraic regions or blocks of formulas and rules are not identified accurately.

6 Discussion and limitations

The objective of this project was to implement and evaluate a prototype system for exploring mathematical formulas in Portuguese for people with visual disabilities, with resources for exploring formulas with abstraction at different levels.

The study was based on a previous study, with usability tests involving the implementation of the adapted ChromeVox add-on (Abreu et al., 2019). It was possible to identify once again problems involving ambiguity in reading formulas, and to explore implications for the design of resources to explore and navigate internal elements of mathematical formulas.

The approach adopted by the adapted ChromeVox (Abreu et al., 2019) directly mapped shortcut keys to structures such as roots, summations, and fractions. This feature provided faster access to elements. However, covering all mathematical structures would become impractical, as it would overload the user's memory to memorize all the key combinations, as demonstrated in a similar case in the study by Stevens et al. (1997). The use of ChromeVox (the only open-source code available at the time) as a basis for implementations was also a limitation for use by visually impaired people. ChromeVox can only be used within the Google Chrome browser. This would require visually impaired users to switch between two screen readers, which could cause overload.

At the time of implementation, however, ChromeVox was the only open-source screen reader that allowed the implementations to be evaluated. Based on these observations, a new add-on was implemented, Access8Math-NavMatBR software, using the NVDA reader. In 2018, Access8Math for reading mathematical formulas was published by (Tseng, 2018), and the source code was made available to the community. With the implementation and testing of the new add-on, it was possible to verify an improvement in resolving the problem of ambiguity in reading, thanks to the abstraction of complex content.

In addition to the improvement in comprehension, it was possible to observe that the application of abstraction of complex contents also helped to reduce memory overload, since the information transmitted to users is presented in the form of chunks delimited contextually by the mathematical structural elements being explored at the moment. Regarding memory overload, Stevens et al. (1997) state that this can be caused by the user's need to memorize many shortcut keys.

Previous tests with the adapted version of ChromeVox (Abreu et al., 2019) showed that users were not able to memorize the implemented shortcut keys (one for each type of mathematical element). Thus, the Access8Math-NavMatBR software proposed to use the default keys of NVDA, having implemented only a new key combination - referring to the activation of abstract reading. The use of the default keys of the screen reader in exploring the formulas facilitated memorization and comprehension.

The advances obtained are important to direct new studies related to manipulation of mathematical formulas using contextual exploration of structural elements and abstraction of complex elements. This approach, initially presented by Stevens et al. (1997), had been for a long time without being the subject of research for new studies on its effectiveness. However, recent works such as that of Fajardo-Flores and Archambault (2014), evaluated by teachers with and without visual impairments, carried out in English, have returned to discuss the use of abstraction of complex contents and contextual exploration. No studies had been found with the application of this approach with Brazilian Portuguese. The results show that it is possible to establish improvements in the comprehension of mathematical formulas using this approach with Portuguese in the Brazilian context.

Soiffer (2015) also used the contextual exploration strategy in version 4 of his MathPlayer system. However, with regard to reading math on the web, that add-on is only capable of reading in Internet Explorer and Mozilla Firefox. According to MathPlayer's current documentation[5], the add-on supports 15 different languages but does not support Portuguese.

The tests also showed the absence of a standard narration pattern for mathematical formulas. Participants suggested both the creation of support material on the reading pattern adopted by the Access8Math-NavMatBR software add-on, as well as the definition of a formula writing standard. The results of a previous study conducted with teachers who worked with visually impaired students (de Lima et al., 2019) showed that there is indeed no standardization in the way teachers narrate this type of content.

The addition of the teaching scenario in the context of numerical analysis provided important insights into how the contextual exploration strategy could be employed in real-world scenarios. The scenario also provides important insights into how the strategy can be used by teachers in classroom. However, the current version of the add-on does not support all strategies that are needed in such cases without the intervention of teachers and support workers. Providing automated support for these tasks could be the focus of future studies.

As a limitation, the Covid-19 pandemic caused significant changes in the testing plan. The tests that were previously planned to be conducted in a laboratory were conducted remotely on the participants' own computers. Therefore, it was not possible to conduct tests with people who have less experience with screen readers and would have difficulty installing the add-on. For this reason, the profile of participants was limited to those with extensive familiarity with the NVDA screen reader.

Although each participant had to install the system on their own computer for testing, there were advantages to being able to use the screen reader with their own reading preferences, such as speed and voice. This fact made it possible to observe issues such as the ease of installation of the add-on, as well as the difficulty in understanding the delimitation between numerator and denominator due to the insufficient pause applied in this separation, combined with the user's own screen reader speed. Installing the add-on in developer mode posed difficulties to participants to install the current version. Making the add-on available as part of the official release from NVDA would make the installing process easier.

The results from the evaluations showed that participants had improvements in the understanding of the formulas However, we noticed important problems that still need more investigation. Helping participants identify where they are at in a given formula, as they zoom in and out, is still a challenge. For first-time users, identifying how to activate the abstract navigation is also an issue, as some participants did not always remember how to activate it.

Many of the identified problems can be related to the design of the tool and may be investigated by proposing improvements to the interaction. Other issues involve difficulties encountered with mathematical content and gaps in mathematical learning. We believe the improvement of such issues will take time, as joint efforts of improvement in assistive technologies and educational approaches advance in parallel.

The people who participated in the tests had completed their primary and secondary education more than five years prior to the tests. This time gap may have had an impact on the tests, which could have yielded different results with students currently enrolled in those levels of education.

7 Conclusions and future work

The objective of this work was to implement and evaluate a strategy for exploring mathematical formulas on the web for people with visual impairments using screen readers.

The results obtained in the tests involving the Access8Math-NavMatBR software add-on showed that, according to what was reported by the participants and observed in the results, contextual exploration can be capable of reducing reading ambiguity by isolating the narration of the formula in semantically delimited sections and, as a consequence of this semantic isolation, also enable a reduction in memory overload. Through the tests, it was found that the use of generic shortcut keys, accompanied by the use of standard reader keys, facilitated learning and memorization by the participants.

As a contribution, this work presents results with implications for the design of systems for reading mathematical formulas in the Brazilian context and on the patterns of exploration in mathematical formulas made by screen reader users. The extended version also presents an analysis with practical examples of the application of the strategy in the context of a higher-education discipline with complex formulas that can pose challenges to students with visual disabilities.

The main limitation of the work resided in the selection of participants and usability testing. Due to the social distancing measures imposed by the COVID-19 pandemic, it was only possible to select participants with a high degree of experience with screen readers and users with different levels of experience.

As future work, the team is in dialogue with the NVDA translation team for the aggregation of terms used in this version, and subsequent submission for pull-request of the implementation of the exploration strategy for use in NVDA.

[5]Available at: https://www.dessci.com/en/products/mathplayer/tech/credits.htm

Studies are also planned with a focus on educational use in teaching mathematics at different levels, with the application of examples such as those presented in the teaching scenario in different disciplines.

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