# Fraud Alerts in Public Health Based on Audit Trails

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**Abstract** Fraud detection and prevention in the public health sector are essential for preserving resource integrity and service quality. Fraud in this context can include financial misappropriations and data manipulation, which not only waste public funds but also compromise healthcare quality and accessibility. This article introduces an audit trail-based approach to identify and rank potential fraud cases involving public health employees in Brazil. Our ranking system highlights cases with higher fraud likelihood based on the analysis of suspicious activity patterns and travel records. Extending prior work, we also introduce three additional analyses on the alerts: temporal evolution, geographical distribution, and occupation-specific trends. Our results enable auditors to trace trends, regional irregularities, and occupation-specific risks, thus enhancing the accuracy and impact of fraud detection strategies.

Keywords: Fraud Detection, Audit Trails, Public Health, Ranking Approach, E-government

## 1 Introduction

Identifying and preventing fraud in the public sector, especially in the health context, is extremely important to ensure the integrity of resources and the quality of services offered to the population [Neubauer *et al.*, 2022]. Fraud in the public sector can take many forms, from misappropriation of financial resources to manipulation of data and information [Kratcoski, 2018; Pereira *et al.*, 2022; Silva *et al.*, 2023]. Such fraudulent practices not only represent a waste of public resources but can also compromise access to and the quality of health services available to citizens.

In the specific context of health, in which the demand for services is often high and the resources are limited, efficient human resource management is crucial. In this regard, the National Registry of Health Establishments, or CNES,<sup>1</sup> was created in 1999 to concentrate information about all health establishments (public, contracted and private) that provide any type of health care service in Brazil. Therefore, this system is an important tool to support decision-making and action planning by public managers, enabling efficient management of financial resources [Pelissari, 2019].

However, even with systems such as CNES, the health sector still faces challenges related to human resource management and data integrity [Rocha *et al.*, 2018]. Problems such as excessive workloads and considerable travel distance between health establishments may conflict with compliance with the workload in all establishments where a professional works, indicating potential fraud. Such practices represent ethical and legal violations, including misappropriating public funds and administrative misconduct.

Given this scenario, it is essential to develop effective strategies for detecting and preventing fraud in the public health sector, thus ensuring transparency and efficiency in the management of resources allocated to health [Kratcoski, 2018; Kumaraswamy *et al.*, 2024]. In the Brazilian context, the growing demand for health services and the challenges faced in effectively managing public resources highlight the urgent need for innovative approaches to combat fraud and corruption. Thus, several studies have focused on analyzing the reliability of data from CNES and its crucial role in the management of medical and hospital equipment [Rocha *et al.*, 2018; Pelissari, 2019].

In partnership with the Public Prosecutor's Office of the State of Minas Gerais (MPMG), this work proposes an approach based on audit trail modeling to identify and rank fraud alerts involving public health employees. Audit trails are sequences of steps to identify specific types of irregularities in government data [Oliveira *et al.*, 2023]. Our ranking system aims to guide audit efforts toward cases with a higher probability of fraud, and the proposed audit trails analyze suspicious patterns in the activity and travel records of public employees.

This article extends the paper presented at the 12th Workshop on Applied Computing in Electronic Government (WCGE 2024) [Dutra *et al.*, 2024]. As a novel contribution, we go further in the characterization of our results by presenting three additional analyses of the alerts generated by the audit trails: (i) temporal evolution, (ii) geographical distribution, and (iii) alerts by occupation. Such analyses allow a deeper understanding of the potential irregularities and provide valuable insights into patterns of fraudulent activities. Indeed, such analyses enable auditors to identify trends over time, detect regional hotspots of irregularities, and as-

<sup>&</sup>lt;sup>1</sup>Acronym in Portuguese for *Cadastro Nacional dos Estabelecimentos* de Saúde: https://cnes.datasus.gov.br/. Access on 05 June 2025.

sess which occupations are more prone to fraud, thus improving the precision and effectiveness of fraud detection efforts.

This article is structured as follows. Section 2 discusses related work, while Section 3 describes the data source considered in the work. Section 4 presents the methodology used in the construction of the audit trails and in the ranking. Section 5 characterizes the results of the trails and the ranking, presenting real examples in which fraud alerts were generated. Next, Section 6 presents a more in-depth analysis of the alerts considering different perspectives. Finally, Section 7 concludes this work, in addition to describing possible limitations and suggesting future work.

## 2 Related Work

Combating fraud in the public sector is a global priority and is essential to ensure the integrity of government institutions [Silva et al., 2020]. A common aspect of the studies that address this task is the need to deal with large volumes of data, generally using artificial intelligence and machine learning to process them [Mongwe and Malan, 2020; Handoko and Rosita, 2022]. In the Brazilian context, recent studies investigate fraud detection procedures and the impact of technologies on anti-fraud awareness in the public sector [Silva et al., 2020]. In addition, Oliveira et al. [2021] analyze the effectiveness of the tools of the Comptroller General of the Union (CGU) in combating fraud during the COVID-19 pandemic.

Public procurements represent an important point of fraud detection in Brazil, and they are the target of several studies that propose solutions to optimize such an investigation. For example, Brandão *et al.* [2024] propose a semi-automatic pipeline for detecting fraud in public procurement processes by combining data quality and machine learning techniques. Other studies use other techniques, including social network analysis [Pereira *et al.*, 2022], heuristics [Oliveira *et al.*, 2022], artificial intelligence and statistical analysis techniques [Pierotti *et al.*, 2024].

Indeed, statistics have been extensively used in developing techniques for fraud detection and analysis. For instance, the works of Silva *et al.* [2023] and Silva *et al.* [2024] propose a specific methodology for overpricing detection on procurement items based on the Interquartile Range, resulting in a system [Costa *et al.*, 2024] to facilitate consultation and analysis of overpricing in public procurement items. In addition, Braz *et al.* [2023] and Braz *et al.* [2024] focus on detecting irregularities in procurement processes specifically related to small companies, mainly because such companies follow specific legislation regarding their annual revenue and their potential links to other bigger (i.e., controller) companies.

Other areas of public administration, including health, have also received attention in the context of e-government [Neubauer *et al.*, 2022; Domingues *et al.*, 2021; Monteiro *et al.*, 2023; Kumaraswamy *et al.*, 2024]. Kratcoski [2018] argues that the public health sector presents numerous opportunities for fraud and corruption, which often challenges the effectiveness of oversight in preventing such crimes, both by service providers and beneficiaries. In Brazil, many studies have been dedicated to analyzing the reliability of data from the National Registry of Health Establishments (CNES) and

its fundamental role in the management of medical and hospital equipment [Rocha *et al.*, 2018; Pelissari, 2019].

Rocha et al. [2018] compared the reliability of a group of data registered in the secondary databases of the CNES. Although cases of inconsistency were identified, the results generally indicated good reliability of the data from the National Registry of Health Establishments, regarding the categories compared. Similarly, the study by Pelissari [2019] revealed that the CNES system is a reliable tool for effective and efficient management. However, managers must be aware of keeping it updated and ensuring the accuracy of the information entered into the system.

Despite the existence of studies analyzing the health sector, there are still few studies that focus specifically on fraud detection in this area. The lack of scientific research in this specific context reveals an opportunity to fill this knowledge gap, especially considering that fraud and irregularities can affect the quality and availability of health services. Therefore, this work seeks to add knowledge in this area by proposing approaches to identify fraud alerts involving public servants in the health area. Our audit trail and ranking approaches, as well as our in-depth characterization analysis, represent a step further in using computing techniques for combating fraud and corruption in the public sector.

# 3 National Registry of Health Establishments (CNES)

The National Registry of Health Establishments (*Cadastro Nacional de Estabelecimentos de Saúde* – CNES) is the Brazilian Ministry of Health's main information system for registering and maintaining information on all health establishments, including public, contracted, and private networks. The system aims to register and update information on health establishments, make data available to other systems, provide information to society on the availability of services, and support decision-making, planning, and programming in the health area. The CNES is publicly available for consultation<sup>2</sup> and downloads.<sup>3</sup>

In general, CNES contains diverse information on health professionals and establishments. Regarding professionals, it provides information such as name, National Health Card (CNS), employment relationships, working hours, and occupations (according to the Brazilian Classification of Occupations – CBO). Regarding establishments, it is possible to obtain the location, trade name, corporate name, National Registry of Legal Entities (CNPJ), type of management, and legal nature, among others.

# 4 Methodology

This section details the methodology used to identify potential frauds involving health workers. Its main steps are based

<sup>&</sup>lt;sup>2</sup>https://cnes.datasus.gov.br/pages/consultas.jsp. Access on 05 June 2025.

<sup>&</sup>lt;sup>3</sup>https://cnes.datasus.gov.br/pages/downloads/arquivosBaseDados.jsp. Access on 05 June 2025.

on building audit trails, a concept that was introduced in Section 1. The steps are outlined in Figure 1 and include: defining the trails (Section 4.1), preprocessing the data (Section 4.2), formal modeling of the trails (Section 4.3), and a ranking process to consolidate the fraud alerts generated by each trail (Section 4.4). Each of these steps is detailed in the following sections.

#### 4.1 Audit Trail Definition

In this work, we define a set of audit trails in collaboration with experts from the MPMG to generate alerts that indicate potential fraud involving public employees in the health area. The trails were designed to cover a variety of suspicious behaviors, considering the complexity and diversity of the activities performed by health professionals. Table 1 presents our five trails, each covering different aspects and information relevant to the links and activities of the employees.

Note that analyzing a single trail alone does not necessarily suggest that fraud has occurred. For example, trail  $T_3$  aims to identify healthcare workers with multiple employment relationships in the same month. According to current Brazilian legislation, this situation is not necessarily illegal. However, by combining this alert with others generated by different trails, it is possible to identify more complex patterns that may indicate the need for investigation by specialists. In other words, creating and executing trails does not replace manual analysis of these cases, but it does improve and speed up auditors' work, guiding their efforts to areas with greater potential for irregularities.

#### 4.2 Preprocessing

After defining the audit trails, the preprocessing process begins after obtaining the query results in the CNES database. Then, we define a field that informs the employee's type of contract. This processing is performed from the establishment *legal nature* field, which stores codes based on the Legal Nature Table of the Brazilian Institute of Geography and Statistics (IBGE).<sup>4</sup> From such codes, it is possible to classify the employee type of contract as public or private.

Furthermore, we calculate the maximum distance between two cities among those in which each public employee operates. To do so, we use the Open Source Routing Machine (OSRM),<sup>5</sup> a tool that allows calculating routes between different locations. This is essential for this work since CNES provides information about the cities in which each employee works during each month but does not include details about the weekly routine or the specific routes they traveled.

However, due to the lack of detailed data on the weekly routes of the employees, the preprocessing uses a simplified approach. Therefore, to estimate the weekly distance traveled by each employee, we consider the scenario that most benefit the employee, assuming that the route between all cities is the route between the two most distant ones. Although this may underestimate the actual distance traveled,

it allows an initial analysis of the audit trails based on available information, helping to identify possible cases of irregularities in the performance of health employees.

## 4.3 Audit Trail Modeling

This section presents the audit trail modeling, as well as the attributes required for each one. The definition of the trails is based on set theory, as illustrated by Figure 2. The modeling uses three base sets that represent the entities of interest: Professional (P), Month (M), and Occupation (O). These sets and their respective formal definitions are detailed below.

- P = {p | p ∈ Professionals} is the set of all professionals present in the CNES database, including those who work in the public and private networks.
- $O = \{o \mid o \in Occupations\}$  is the set of all occupations performed by health workers (e.g., cardiologist, nurse, physiotherapist).
- $M = \{m \mid m \in [01/2015, 10/2023]\}$  is the set of all months between January 2015 and October 2023.

From the Cartesian product of the base sets, four new sets called  $Cartesian\ sets$  are created. These sets represent different attributes of the following entities: weekly working hours (H), maximum weekly distance (D), number of private contracts (I), and number of public contracts (U). The formal definition of each is presented below.

- $H = P \times M \times O = \{h_{pmo} \mid h_{pmo} \in \mathbb{R}^+ \land p \in P \land m \in M \land o \in O\}$  is the set of weekly working hours that professional p worked during month m in occupation o.
- $D = P \times M = \{d_{pm} \mid d_{pm} \in \mathbb{R}^+ \land p \in P \land m \in M\}$  is the set of maximum weekly distances that professional p traveled during month m.
- $I = P \times M = \{i_{pm} \mid i_{pm} \in \mathbb{R}^+ \land p \in P \land m \in M\}$  is the set of private employment contracts that professional p had in month m.
- $U = P \times M = \{u_{pm} \mid u_{pm} \in \mathbb{R}^+ \land p \in P \land m \in M\}$  is the set of public employment contracts that professional p had in month m.

In the context of the proposed modeling, an audit trail refers to a set of rules designed to identify fraud alerts among public health employees. The set of audit trails is denoted by  $T = \{T_1, T_2, T_3, T_4, T_5\}$ , in which  $T_x = \{(p, \hat{M}_{xp}, a_{xp})\}$  represents an ordered triplet that contains: the professional (p) who violated these rules, the months  $(\hat{M}_{xp})$  in which these violations occurred for this professional, and the alert  $(a_{xp})$  indicated by trail x for professional p. For the elements of this ordered triplet, the following constraints are applied: (i)  $p \in P$ ; (ii)  $\hat{M}_{xp} \subset M$ ; (iii)  $|\hat{M}_{xp}| > 0$ ; (iv)  $\forall (m \in \hat{M}_{xp}) \rightarrow u_{pm} > 0$ ; (v)  $a_{xp} \in \mathbb{R}^+$ ; (vi)  $a_{xp} > 0$ .

Table 2 describes the rules that  $\hat{M}_{xp}$  must meet for a given professional p to be classified in each of the trails, as well as the formula to calculate the alert  $(a_{xp})$  for each trail. This alert indicates how far the professional is from the established norm. For example, trail  $T_5$  identifies employees with a very high weekly workload. Thus, we use a previously

<sup>&</sup>lt;sup>4</sup>https://concla.ibge.gov.br/estrutura/ natjur-estrutura/natureza-juridica-2021. Access on 05 June 2025.

<sup>&</sup>lt;sup>5</sup>OSRM: https://project-osrm.org/. Access on 05 June 2025.

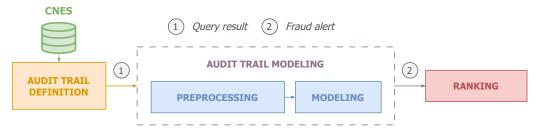


Figure 1. Methodology for detecting fraud alerts involving public employees.

Table 1. Audit trails definition.

#	Trail Name	Rule
$T_1$	Long distance traveled	Check for healthcare workers who travel a long distance, weekly, between the cities where they work.
$T_2$	Very long working period	Check for healthcare workers with very long working periods (in hours).
$T_3$	Multiple public contracts	Check for healthcare workers who have multiple public contracts in the same month.
$T_4$	High number of employment contracts	Check for healthcare workers with many employment contracts in the same month.
$T_5$	High weekly workload	Check for healthcare workers who have a very high weekly workload, taking into account the total hours worked and an estimate of the travel time between cities.

established threshold for this workload, and the more this limit is exceeded, the greater the penalty for this trail. Furthermore, the penalty increases according to the number of months the employee maintains this high workload.

The function  $f(d_{pm})$  in trail  $T_5$  calculates the average time required to travel the distance  $d_{pm}$  between cities. Furthermore, in trail  $T_1$ , the variable  $threshold\_D_m$  represents a dynamic threshold derived from a statistical approach, using the maximum weekly distance of all professionals over the time period. Initially, the data were grouped by year, and then the interquartile range (IQR) of the distances in each year was calculated. This value was then multiplied by a scaling factor of 1.5, as shown in Equation 1, where  $Q_{3m}$  and  $Q_{1m}$  represent, respectively, the third and first quartile of the maximum weekly distances in the year of month m.

threshold 
$$D_m = Q_{3m} + 1, 5(Q_{3m} - Q_{1m})$$
 (1)

The thresholds for the other trails were established in collaboration with MPMG specialists. In trail  $T_2$ , the objective is to identify situations in which the employee has a weekly workload of more than 60 hours for a single occupation. In trail  $T_3$ , we aim to identify cases where the employee has more than three public employment contracts in the same month. For trail  $T_4$ , the aim is to detect situations where the employee has more than ten total employment contracts in the month. Finally, in trail  $T_5$ , the objective is to identify cases where the total weekly workload, added to an estimate of the distance traveled between cities, exceeds 126 hours per week. This value of 126 hours is equivalent to a workload of 18 hours per day, seven days a week.

It is important to emphasize that all professionals in the CNES database were considered in the modeling, but the trails seek to find alerts only for public employees. This is based on the assumption that even if a professional is classified in several trails, if they only have private ties, they would not be causing harm to public assets. Therefore, it is not the goal of this work to perform an internal audit in

private companies. Thus, professionals who have at least one public contract are considered public employees (i.e.,  $\forall (m \in \hat{M}_{xp}) \rightarrow u_{pm} > 0$ ).

In addition to the description of the rules of  $\hat{M}_{xp}$ , Table 2 presents the formula of the alert  $a_{xp}$  for each trail. This calculation consists of the sum of the difference between the variable of interest and the threshold, considering each month the professional failed to comply with the trail rule. Before being added, the difference is normalized on a scale from 1 to 100 (indicated by the norm operator) to prevent one path from dominating the others due to different orders of magnitude of the variables of interest. For example, in path  $T_1$ , the variable of interest is the distance, measured in kilometers, while in  $T_3$  it is the number of public contracts, resulting in different orders of magnitude.

## 4.4 Ranking

This section outlines the strategy used to rank healthcare employees potentially involved in fraudulent activities. For each employee, an alert indicator is computed by aggregating the individual results of multiple audit trails, each weighted according to a predefined risk level. Table 3 presents the empirically defined risk levels for each trail, established in collaboration with expert auditors based on the severity of the alerts typically generated. To calculate the alert indicator, each trail x receives a numerical weight  $w_x$  according to its risk level, according to Equation 2.

$$w_x = \begin{cases} 1; & \text{if } x \in \{3\} \\ 0, 1; & \text{if } x \in \{4, 5\} \\ 0, 01; & \text{if } x \in \{1, 2\} \end{cases}$$
 (2)

In short, trails classified as high risk receive a weight of 1, those classified as medium risk have a weight of 0.1, and those classified as low risk have a weight of 0.01. Thus, it is possible to define the fraud alert indicator  $R_p$  of a professional p as the sum of the products between the alerts  $a_{xp}$ 

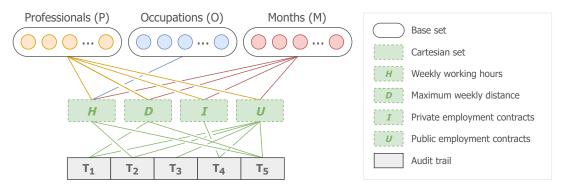


Figure 2. Modeling audit trails based on set theory.

Table 2. Formal definition of the rules for each audit trail.

#	Rules of $\hat{M}_{xp}$	$ig $ Formula to calculate $a_{xp}$
$T_1$	$\hat{M}_{xp} = \{m \mid \forall m \rightarrow d_{pm} > threshold\_D_m\}$	$a_{xp} = \sum_{m}^{ \hat{M}_{xp} } (d_{pm} - threshold\_D_m)_{norm}$
$T_2$	$\hat{M}_{xp} = \{ m \mid \forall m \to (\exists o \to (h_{pmo} > 60)) \}$	$a_{xp} = \sum_{m}^{ \hat{M}_{xp} } \left[ \sum_{o}^{ O } (h_{pmo} - 60)_{norm} \right]$
$T_3$	$\hat{M}_{xp} = \{ m \mid \forall m \to u_{pm} > 3 \}$	$a_{xp} = \sum_{m}^{ \hat{M}_{xp} } (u_{pm} - 3)_{norm}$
$T_4$	$\hat{M}_{xp} = \{ m \mid \forall m \to (u_{pm} + i_{pm}) > 10 \}$	$a_{xp} = \sum_{m}^{ \hat{M}_{xp} } (u_{pm} + i_{pm} - 10)_{norm}$
$T_5$	$\hat{M}_{xp} = \left\{ m \mid \forall m \to \left( \sum_{o}^{ O } h_{pmo} + f(d_{pm}) \right) > 126 \right\}$	$a_{xp} = \sum_{m}^{ \hat{M}_{xp} } \left[ \left( \sum_{o}^{ O } h_{pmo} + f(d_{pm}) \right) - 126 \right]_{norm}$

Table 3. Risk level of each trail.

Risk	Trail
High	$T_3$ - Multiple public contracts
Medium	$\left  \begin{array}{c} T_4 \text{ - High number of employment contracts} \\ T_5 \text{ - High weekly workload} \end{array} \right $
Low	$\left  \begin{array}{c} T_1 \text{ - Long distance traveled} \\ T_2 \text{ - Very long working period} \end{array} \right $

generated by each trail x and its corresponding weight. Formally, this indicator is expressed by Equation 3.

$$R_p = \sum_{x=1}^{5} \left( a_{xp} \times w_x \right) \tag{3}$$

Once the alert indicator  $R_p$  has been calculated for each professional p, we produce the ranking by ordering the professionals based on their alert indicator in decreasing order. This generates the set  $\overline{P}$  representing the professionals ranked by the alert indicator.  $\overline{P} \subseteq P$  e  $\overline{P} = \{p \mid \forall p \rightarrow R_p > R_{(p+1)}\}$ .

## 5 Experimental Analysis

To evaluate the proposed methodology, we perform an experiment considering a specific dataset of public health workers, as detailed in Section 5.1. Then, in Section 5.2, we characterize the results of the proposed trials and analyze them to

deepen their understanding. Finally, in Section 5.3, we complement the experimental analysis by examining the characteristics of the ranking process of workers based on fraud alert indicators.

#### 5.1 Data

For this experimental analysis, we consider a dataset of health workers in the Brazilian state of Minas Gerais (MG). The data were filtered to include only health workers with an employment contract in the municipalities of this state, and they were obtained from CNES from January 2015 to October 2023. Therefore, the final dataset contains 838,443 workers distributed across the 853 municipalities of MG.

## 5.2 Trail Results

Here, we analyze the results obtained from the proposed trails in the considered dataset. Table 4 presents the characterization of the alerts generated by the trails, including their risk level, the total number of employees involved, their minimum, maximum, and average values, as well as the sum of the alerts of all employees involved. The employees are counted based on their respective National Health Card (CNS) numbers, which are a unique identification of each citizen in the Brazilian Public Healthcare System (SUS).

Regarding the number of suspicious employees identified by each trail, trail  $T_1$  presents a relatively higher number

Trail	Risk	#P	$\min(\mathbf{a_{xp}})$	$\max(\mathbf{a_{xp}})$	$\bar{a}_{xp}$	$\sum \mathbf{a_{xp}}$
$T_1$	Low	9,710	1	6,205.75	323.37	3,139,994.97
$T_2$	Low	1,776	1	985.79	42.57	75,607.42
$T_3$	High	1,562	1	1,950	49.06	76,630
$T_4$	Medium	2,806	1	3,481	167.89	341,834
Tr.	M - J:	2 (02	1	520.1	22.02	115 206 74

**Table 4.** Characterization of the results of the trails.

**#P:** employees Quantity  $\mathbf{min}(\mathbf{a_{xp}})$ : Minimum alert  $\mathbf{max}(\mathbf{a_{xp}})$ : Maximum alert  $\mathbf{\bar{a}_{xp}}$ : Average alert  $\sum \mathbf{a_{xp}}$ : Sum of alerts

compared to the others. This trend is corroborated when considering the average number of alerts assigned, where trail  $T_1$  also stands out for presenting a significantly higher value than the others. For example, the average number of alerts for  $T_1$  is at least twice as high as that of most trails. Such values suggest a higher concentration of alerts involving the distance traveled by the employees. In addition, the high number of cases identified by  $T_1$  may also be related to the threshold used in this trail,  $threshold\_D_m$ , which is established dynamically by a statistical approach (Equation 1), unlike the other trails that have a fixed threshold.

Regarding trail  $T_3$ , classified as having the highest risk among all, despite having fewer suspicious employees compared to  $T_1$ ,  $T_4$ , and  $T_5$ , the average number of alerts assigned is still significant. This suggests that, although the absolute number of suspects is smaller, the irregularities detected in this trail tend to be more severe or impactful. This analysis reinforces the importance of considering not only the quantity but also the severity of the irregularities detected when assessing the risk associated with each audit trail.

In general, no significant relationship is observed between the trail's risk level and the average number of alerts. For example, trails  $T_1$  and  $T_2$ , classified as having low risk, have very different average alerts. While  $T_1$  has an average of  $\bar{a}_{1p}=323.27$ , trail  $T_2$  has  $\bar{a}_{2p}=42.57$ , which is the second lowest value observed among all trails. On the other hand, trail  $T_3$ , classified as high risk, has an average of alerts  $\bar{a}_{2p}=49.06$ . This lack of association between the risk level and the average of alerts suggests that other factors may influence the magnitude of the irregularities detected in each trail.

To further analyze the relationship between trails beyond their risk level, we analyze the correlation between the alerts generated by the trails for the considered employees. In this analysis, we use the Pearson coefficient (r), which measures the linear correlation between two variables. All resulting correlation coefficients are in the interval [-0.1;0.2], indicating a weak correlation between them. This result suggests that variation in one variable does not cause significant changes in the others, and each trail contributes distinct information to the analysis.

#### 5.3 Ranking Results

We now present and analyze the results of the employee ranking proposed in Section 4.4. Initially, we present a qualitative analysis of the top 10 employees classified using the proposed approach. Then, we analyze the influence of each track on the task result using a ranking quality measure.

**Qualitative Analysis.** Table 5 presents the top 10 employees ranked according to the fraud alert indicator  $(R_p)$ .

The columns represent the ordered employees, and the rows present the alerts  $a_{xp}$  generated by the trails  $T_x$ . The last row contains the employee's fraud alert indicator.

Overall, the ranking analysis reveals that the top-ranked employees have high alert values  $a_{xp}$  for trails  $T_1$  (high distance traveled),  $T_3$  (multiple public contracts), and  $T_4$  (high number of employment contracts). Although the trail  $T_1$  has a low risk and  $T_4$  a medium risk, such employees were still well ranked because they have high alert values in these trails combined with alerts in trail  $T_3$ , which is high risk.

The first three employees in the top 10 have a consistent pattern of alerts. They all have a significant amount of alerts in trail  $T_3$ , high alert values for trails  $T_1$  and  $T_4$ , and considerable alerts in trail  $T_5$  (high weekly workload). Furthermore, none of the top-ranked employees have alerts for trail  $T_2$  (very long working hours). Such patterns suggest the existence of similar behaviors among the identified employees, which requires more detailed investigations by the auditors.

**Influence of Trails.** To evaluate the influence of each trail in the ranking, we use Discounted Cumulative Gain (DCG), a ranking quality metric [Wang *et al.*, 2013]. DCG considers the sum of the gains  $c_{px}$  weighted by the position of each employee in the ranking. Here, the gain  $c_{px} = a_{xp} \times w_x$  represents the influence of trail x on the risk indicator of employee p. Equation 4 presents the formula to calculate the DCG, where  $\overline{P}$  is the set of ranked employees and p is the employee's position in the ranking. This metric allows for a more accurate assessment of the contribution of each track to the overall risk indicator.

$$DCG_x = \sum_{p=1}^{|\overline{P}|} \frac{c_{px}}{\log_2(p+1)} \tag{4}$$

Figure 3 presents the DCG values of each trail. The results suggest a relationship between the DCG value and the risk associated with each trail. For example, trail  $T_3$ , classified as high risk, exhibits a significantly higher DCG value than trails with lower risk. This suggests that higher-risk trails have a greater influence on the ranking process, contributing more to identifying employees with potential irregularities.

However, there is an exception in the result for trail  $T_1$ . Despite being classified with a low risk level, it has a higher DCG than trail  $T_5$ , which has a medium risk level. This is because trail  $T_1$  generated alerts for a substantially larger number of employees compared to  $T_5$  (according to Table 4). Therefore, even with a low risk level,  $T_1$  is a more influential trail than  $T_5$  in the ranking process due to its more significant impact on identifying potential irregularities.

	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
$a_{1p}$	3,021.46	487.20	445.63	313.13	921.42	3,869.01	0	1,513.63	0	0
$a_{2p}$	0	0	0	0	0	0	0	0	0	0
$a_{3p}$	1,950.00	1,656.00	1,113.00	990.00	1,003.00	967.00	990.00	827.00	879.00	746.00
$a_{4p}$	874.66	692.66	379.66	762.33	441.00	120.00	84.00	735.00	91.66	92.00
$a_{5p}$	72.07	20.19	10.61	0	0	21.60	0	0	0	35.38
$\mathbf{R}_{\mathbf{p}}$	2,074.89	1,732.16	1,156.48	1,069.36	1,056.31	1,019.85	998.4	915.64	888.16	758.74

**Table 5.** Top 10 employees with highest values for the fraud indicator  $\mathbf{R_s}$ .

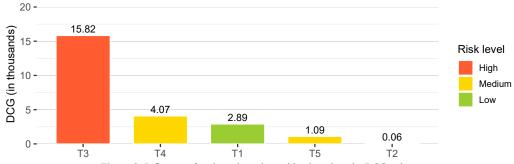
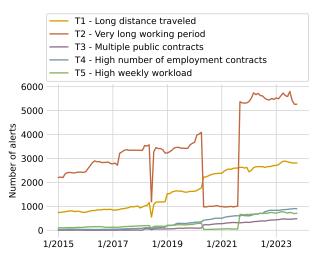


Figure 3. Influence of each track on the ranking based on the DCG value.



**Figure 4.** Time series of the monthly number of alerts generated by each audit trail from January 2015 to October 2023.

## 6 Alert Characterization

Here, we further characterize our results with three additional analyses of the alerts generated by the audit trails: temporal evolution in Section 6.1, geographical distribution in Section 6.2, and alerts by occupation in Section 6.3. These analyses reveal potential irregular activity patterns, enabling the identification of trends over time, regional hotspots of irregularities, and occupations with a higher potential for fraud.

#### **6.1 Temporal Evolution**

The first characterization analysis focuses on the temporal evolution of alerts generated by the audit trails. Figure 4 illustrates the time series of all trails during the eight-year span of the data. Each point in the time series represents the total number of alerts generated for each month. Overall, there is an increase in such a number for all considered audit trails, suggesting an actual rise in fraudulent activities that require further investigation, particularly as the healthcare

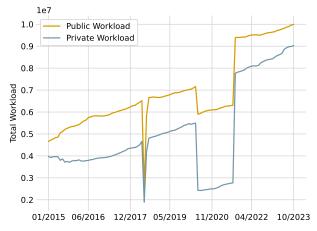


Figure 5. Time series of the total workload separated by public and private from January 2015 to October 2023.

sector navigates increasing complexities and challenges.

Although all trails present the same increasing pattern, trails  $T_1$  (long distance traveled) and  $T_2$  (very long working period) notably have a higher number of alerts during the considered period. This may represent a greater prevalence of irregularities associated with the activities tracked by these specific audit trails. For example, some employees may be overextending their workloads or engaging in questionable travel practices. Such findings raise concerns about potential violations of labor regulations and ethical standards, which could compromise the quality of care provided to patients.

However, it is important to note an abrupt alert drop across all audit trails in June 2018. Specifically, the number of monthly alerts is significantly lower than in May, with values returning to previous levels by July. Such an anomaly suggests a potential gap in data collection or reporting for June 2018, as no seasonal or trend-based factors appear to justify such a sharp decline. The absence of data for this period may affect the overall analysis, particularly if patterns observed in other months are assumed to continue uninterrupted.

A second notable anomaly appears in the abrupt drop in



**Figure 6.** Time series of the total contracts separated by public and private from January 2015 to October 2023.

alerts for trails  $T_2$  and  $T_5$  (high weekly workload) from March 2020 to July 2021. This timeframe coincides with the onset of the COVID-19 pandemic, suggesting that pandemic-related factors may have influenced these trails. Since these audit trails specifically monitor healthcare workers' workloads, one possible explanation is a reduction in recorded workload volume due to shifts in healthcare operations and staff deployment during the crisis. To corroborate this hypothesis, Figure 5 illustrates the time series of total workload volume for healthcare workers, which shows a similar pattern of reduction from March 2020 to July 2021.

Although the average workload decreased during the pandemic, the number of healthcare employees continued to grow, as illustrated in Figure 6. Indeed, there is a noticeable increase in contracts, both public and private, beginning in March 2020. Such a rise aligns with the elevated demand for healthcare professionals during the COVID-19 pandemic, as the need for additional staff to handle the crisis was urgent. This finding underscores the value of contextual data analysis in audit trails, as it allows auditors to distinguish between typical irregularities and those arising from extraordinary circumstances, such as a public health crisis.

Still regarding the average workload decrease, we also do not discard the hypothesis of missing data during this period, since records may have been incomplete or inconsistently updated due to operational disruptions during the pandemic. Such disruptions could have affected data collection processes, as health institutions prioritized emergency responses over routine administrative tasks. Consequently, gaps or inaccuracies in the recorded workloads may have artificially reduced the number of alerts in trails  $T_2$  and  $T_5$ , especially if certain establishments or regions faced significant data reporting delays.

#### 6.2 Geographic Distribution

To understand the geographic distribution of alerts, Figure 7 shows the frequency of alerts across the cities most impacted by the audit trails. Belo Horizonte, the capital of Minas Gerais and its largest city in terms of population and economic activity, emerges as the city with the highest alert frequency. As the capital, Belo Horizonte has the largest

healthcare infrastructure and demand for healthcare workers, which likely increases the probability of generating alerts due to a greater workforce volume and workload requirements.

Moreover, Uberlândia, the state's second-largest city by population and income (according to IBGE), ranks fourth in alert frequency, behind Betim and Juiz de Fora. Previously, Gomide *et al.* [2023] identified Betim as a significant outlier in healthcare expenditure, which might explain its high alert rate. Higher healthcare spending in Betim enables it to hire more healthcare professionals, thereby increasing the likelihood of alerts due to irregularities associated with larger workforces. This elevated alert rate in Betim underscores that cities with high healthcare investments may have more audit trail detections, regardless of their population size.

Furthermore, the same study identified Belo Horizonte, Betim, Uberlândia, Juiz de Fora, Contagem, Montes Claros, and Ipatinga as cities with the highest healthcare expenditures. This pattern aligns with the data in Figure 7, where these cities appear as alert hotspots. The correlation between healthcare spending and alert frequency supports the idea that cities with substantial investments in healthcare infrastructure and workforce are more likely to encounter issues flagged by audit trails.

The cities with the most alerts for the audit trails are also regional healthcare hubs within the state of Minas Gerais, as indicated by the most recent Health Regionalization Master Plan (in Portuguese, *Plano Diretor de Regionalização*)<sup>6</sup>. Due to their extensive healthcare infrastructure and higher concentrations of healthcare professionals, such hubs may naturally produce a greater volume of alerts, reflecting the larger scale and complexity of healthcare operations they manage compared to smaller municipalities.

Overall, the geographic analysis of audit trail alerts reveals that healthcare infrastructure size, workforce volume, and municipal healthcare expenditure are key factors influencing the frequency of alerts. Cities with higher healthcare spending tend to generate more alerts due to the larger and more complex operations they manage, even when they are not the largest cities by population. This pattern suggests that healthcare investment and operational scale may better predict the need for audit oversight than population size alone.

#### 6.3 Occupation Analysis

The last characterization analysis focuses on the occupations most frequently associated with alerts in the audit trails. Figure 8 illustrates the distribution of alerts across the top 10 occupations most frequently flagged by the audit trails. General Practitioners stand out with the highest number of alerts, appearing across all five trails, which suggests their widespread involvement in scenarios where audit trails commonly detect irregularities. This prominence may highlight the critical role that General Practitioners play in the healthcare system, as they often manage diverse patient populations and complex treatment plans, leading to a higher likelihood of encountering situations that trigger alerts.

Resident Physicians follow, with significant alerts primarily concentrated in trail  $T_2$  (very long working period), due

<sup>&</sup>lt;sup>6</sup>https://www.saude.mg.gov.br/gestor/regionalizacao. Access on 05 June 2025.

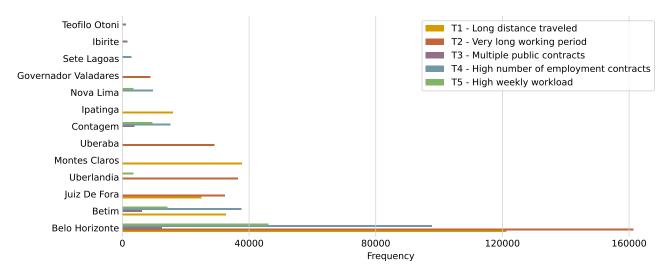


Figure 7. Frequency of alerts for all audit trails for the most affected cities from all data.

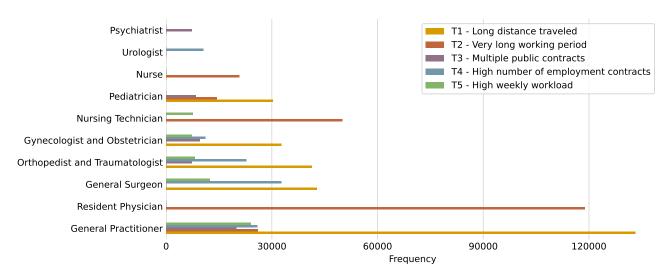


Figure 8. Frequency of alerts across all audit trails for the top occupations affected.

Table 6. Top 10 most frequent occupations in our dataset.

#	Occupation	Count	Percentage
1	Nursing Technician	143,334	12.67%
2	Community Health Agent	85,326	7.54%
3	Nurse	60,521	5.35%
4	General Practitioner	55,501	4.91%
5	Administrative Assistant	41,852	3.70%
6	Nursing Assistant	39,606	3.50%
7	Epidemic Control Agent	29,626	2.62%
8	Receptionist, General	29,422	2.60%
9	General Dental Surgeon	24,028	2.12%
10	Family Health Strategy Doctor	23,691	2.09%

to their 60-hour weekly requirement often leading to alerts when combined with any additional workload. Other occupations, including Nurses, Urologists, and Psychiatrists, show alerts in fewer trails, which aligns with the expectation that certain audit trails are more sensitive to specific work patterns or scheduling behaviors in the healthcare sector.

To understand the occupational context further, Table 6 lists the top 10 most common occupations within the dataset. Notably, Nursing Technicians, Community Health Agents,

and Nurses rank highest by count, reflecting their roles as high-demand professions within the healthcare system. Medical professionals such as General Practitioners are relatively less common in the dataset but still generate the most alerts due to their working patterns, which often include multiple contracts or long-distance travel for work—a trend that the audit trails are designed to capture.

Comparing Figure 8 and Table 6, we note that the presence of alerts is not merely a factor of occupation frequency. Despite lower counts in the dataset, medical professionals like General Practitioners and Resident Physicians dominate alert frequency, emphasizing the trails' focus on occupations prone to specific irregularities, such as excessive hours or multi-location contracts. In contrast, occupations with administrative roles, such as Administrative Assistants or Receptionists, are not frequently flagged, as they generally do not exhibit the patterns these trails target.

In summary, the occupation analysis shows that while nonmedical roles constitute a large portion of the workforce, it is primarily the medical professionals whose work patterns align with audit trail parameters who generate the most alerts. This alignment with expected behaviors reinforces the trails' design to monitor roles where irregularities, especially concerning workload and travel between cities, are more likely to occur. For example, it is common for doctors in Brazil to work in multiple locations, which inherently increases their alert risk. While these working patterns are legitimate under many circumstances, they remain a focus for audit trails to detect potential administrative issues requiring attention.

## 7 Conclusion

This article proposes an approach based on audit trail modeling to identify potential fraud alerts related to public health employees. Such trails are sequences of steps to detect suspicious patterns in the activity and movement records of employees, allowing an efficient and systematic analysis of potential fraud cases. In addition, they were defined based on data from the National Registry of Health Establishments (CNES), a system that gathers information about health professionals and establishments in Brazil.

To validate the effectiveness of the proposed methodology, we conduct an experiment using an extensive dataset of public health employees in the state of Minas Gerais. The results demonstrate that the audit trails can list employees with the highest potential for irregularities, especially when combined using our ranking method. In addition, we go further in our analyses by performing a more in-depth characterization of the alerts, considering three additional analyses: a temporal analysis, a geographic distribution analysis, and an occupational analysis.

Temporal analysis reveals that the COVID-19 pandemic significantly influenced the number of contracts, highlighting data gaps regarding working hours during this period. Geographic analysis indicates that the cities most affected by alerts are regional healthcare hubs with higher healthcare expenditures, suggesting a correlation between increased spending and a greater likelihood of fraud detection. Additionally, our occupational analysis shows that medical professionals, especially General Practitioners, frequently appear in the alerts, aligning with the audit trails' focus on detecting irregularities related to their practices.

Overall, our results suggest that the proposed methodology can serve as a valuable tool for auditors, offering an initial screening analysis given the large volume of data available. Hence, this methodology not only offers an automated way to identify potential fraud among healthcare employees but also highlights the importance of an integrated and multifaceted approach to audit data analysis.

Limitations and Future Work. The main limitation of this work is the dependence on data available in the CNES, which may influence the accuracy and comprehensiveness of the analyses performed. In addition, ethical and legal issues related to the privacy of public employees also need to be considered. In future work, we plan to explore more advanced data analysis techniques and integration with other data sources to improve further the detection and prevention of fraud in the healthcare area. In addition, we intend to perform a manual validation step of the results with experts to verify the accuracy of the proposed trails.

## **Declarations**

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## **Competing interests**

The authors declare that they have no competing interests.

## Availability of data and materials

The enriched datasets and source code used in this study are not publicly available due to privacy and confidentiality agreements established with the funding institution, i.e., the Prosecution Service of the State of Minas Gerais (Ministério Público do Estado de Minas Gerais – MPMG) through the Analytical Capabilities Project (Programa de Capacidades Analíticas), which imposes restrictions on the sharing of data and materials to protect sensitive information.

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