Deep Learning-Driven Parameter Adaptation for Underwater Image Restoration

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Abstract. In this paper we propose a learning-based approach to enhance underwater image quality by optimizing parameters and applying intensity transformations. Our methodology involves training a CNN Regression model on diverse underwater images to learn enhancing parameters, followed by applying intensity transformation techniques. In order to evaluate our approach, we conducted experiments using well-known underwater image datasets found in the literature, comprising real-world subaquatic images and we propose a novel underwater image dataset, composed by 276 images from Amazon turbid water rivers ¹. The results demonstrate that our approach achieves an impressive accuracy rate in three different underwater image datasets. This high level of accuracy showcases the robustness and efficiency of our proposed method in restoring underwater images.

1. Introduction

Research work on underwater image restoration have been increasing in recent years and are extremely important for several applications in subaquatic scenarios. The acquisition process of good quality subaquatic images is a complex operation, representing a significant challenge for visual data capture and analysis, especially, due to the different underwater environments such as oceans, rivers and lakes [Liu et al. 2022a][Lyu et al. 2022][Martinho et al. 2022]. Distinct aspects contribute to the mentioned challenging acquisition, including i) water turbidity, caused by suspended particles; ii) presence of marine organisms, that contribute to the degradation of image quality and water scattering; and iii) uneven lighting and optical distortion, resulting in reduced visibility and loss of details [Qiao et al. 2023].

Considering the underwater image acquisition complexity as mentioned earlier, the underwater image quality enhancement is a challenging problem, since there are many aspects affecting the subaquatic image quality. Additionally, the aforementioned realworld applications demonstrate the relevance of the addressed problem. In order to tackle this problem, advanced algorithms and techniques have been proposed to compensate the adverse effects of light scattering and absorption. By employing such techniques, underwater images can be transformed to reveal fine details, high contrast and precise color

¹The proposed Amazon Underwater Image Dataset (AUID) is available in Github: https://github.com/lauramartinho/Underwater-image-enhancement-based-onfusion-of-intensity-transformation-techniques

representation. Several methods transforming the image intensities can be employed such as color and gamma correction, histogram and contrast adjustment and unsharp enhancement [Zhao et al. 2023].

In this paper we propose a Deep Learning-based approach for underwater image restoration, enhancing the quality and clarity of subaquatic images. We use a Convolutional Neural Network (CNN) regression model to estimate the best parameters to reduce the image degradation. Next a sequence of intensity transformation techniques are applied using the parameters found by the CNN, in order to improve the subaquatic images quality. Experiments were carried out using two well-known underwater image datasets. We have also carried out a comparison of our results with other relevant state-of-the-art algorithms. The obtained results provide evidence supporting the effectiveness of the proposed approach for underwater image restoration, showcasing improved accuracy and significantly clearer underwater images.

Our work offers two main contributions, which can be summarized as follows:

- We propose a Deep Learning-based approach to learn the best parameters in the process of restoring the quality of underwater images. The proposed a regression process acquires knowledge from the different water conditions (like turbidity, low lighting, scattering and distortion in water), enabling the estimation of the better parameters for different underwater images. The proposed restoration approach presents high accuracy even regarding different water conditions;
- We propose a challenging dataset composed by 276 underwater images, acquired in the Urubu river, a blackwater tributary of the Amazon river. The proposed dataset comprises subaquatic images with intense turbidity and low lighting and scattering in the water. To the best of our knowledge this is the first dataset of underwater images from the Amazon region.

Furthermore, the contributions of this work are published in of the main international conferences and journals in the area some [Martinho et al. 2022][Martinho et al. 2023][Martinho et al. 2024].

2. Related Work

Underwater image restoration has been the subject of intense research in the scope of computer vision. This type of approach consists of enhance the quality of subaquatic images to support real-world applications in different scenarios [Zhuang et al. 2022] [Liu et al. 2022a][Gangisetty and Rai 2022].

Several proposed state-of-the-art methods were designed to restore the resolution of images, enhancing visual perception by learning strategies. In [Qiao et al. 2023] the authors proposed an approach based on a deep learning network with multi-Scale and multi-dimensional feature to turbidity, light absorption, and scattering problem by a convolutional and pooling structure. [Huang et al. 2022] presents a two-step strategy based on color restoration and image fusion with deep learning and conventional image enhancement techniques. This work uses an adaptive color compensation method and color restoration. A multi-stage deep Convolutional Neural Networks (CNN) framework with feature reconstruction loss and mean Squared Error is proposed in [Sharma et al. 2022], optimizing it using the traditional pixel-wise and feature-based cost functions. Other work proposed adaptive-learning techniques to remove color casts and low illumination and restore information, such as [Liu et al. 2022a] which proposed the adaptive learning attention network based on supervised learning named LaNet and [Wang et al. 2022] which presented a reinforcement learning method with adaptive underwater presentation characteristics to improve image details.

Convolutional neural networks methodologies are used to enhance underwater images by removing fog and restoring color [Zheng and Luo 2022] using color restoration module, an end-to-end defogging module and a brightness equalization module, using a CNN and an encoder-decoder backbone to color restoration, defogging and brightness equalization. [Liu et al. 2022b] was also based on model image simulation, learningbased image enhancement with CNN and encoder-decoder backbone. However, some CNN approaches do not require encoders to color, contrast or dehaze underwater images, accomplishing improvement through multiscale densely connected deep CNNbased model, underwater optical imaging formulation and data-driven deep learning, as in [Jiang 2022], or methods that uses medium transmission maps to restore real-world underwater images [Kai et al. 2022] involving training a CNN using a GAN framework with dilated residual blocks (DRBs).

Our work proposes a new approach for the enhancement of underwater images, using a combination of processing techniques and convolutional neural networks (CNNs). This strategy aims to overcome the challenges when dealing with the limitations and distortions present in images captured in underwater environments. It is possible to identify several approaches that address the problem in a similar way to ours, including fusion processing techniques [Zhao et al. 2023], widely used to improve the quality and detail of images. It involves combining multiple images of the same scene, processed by different techniques, to produce a single high-quality image. As well as techniques using CNNs, neural networks specialized in image processing [Sharma et al. 2022], capable of learning and extracting relevant features automatically [Liu et al. 2022a]. Widely used in various computer vision tasks in the context of underwater image enhancement, CNNs can be trained to learn specific patterns of distortions and noise present in these images, with the goal of restoring the original quality.

3. Methodology

The proposed methodology is composed by two main steps: i) Learning-based Parameters Estimation; and ii) Intensity Transformation Fusion, as can be seen in Figure 1, while further details will be presented in the next subsections. The first step consists of training a regression convolutional neural network, using a collection of raw underwater images (\mathcal{I}) , to figure out the best parameters (\mathcal{P}) for enhancing the quality of subaquatic images, regarding several scenarios and water conditions. The network analyzes the training image dataset and estimates the most efficient parameters for enhancing underwater images. Secondly, based on the identified optimal parameters, obtained from the learning process, intensity transformation functions are applied to restore the subaquatic images, resulting in restored underwater images (\mathcal{R}) .

3.1. Learning-based Parameters Estimation

The CNN model presents three convolutional layers with 64, 128, and 128 filters, utilizing (3x3) filters, ReLu activation, and (2x2) Max Pooling for feature extraction, followed



Figure 1. Overview of the proposed approach for subaquatic image restoration.

by batch normalization for efficiency. It includes two fully-connected layers with 128 and 64 units and 0.5 rate Dropout layers to combat overfitting. The output layer has 4 neurons for estimating parameters like color correction and gamma correction intensity. Training aims to minimize MSE using Adam with a 0.001 learning rate over 50 epochs and a batch size of 128. This architecture and training strategy, detailed in our journal [Martinho et al. 2024], are chosen for their effectiveness in underwater image parameter estimation, demonstrating adaptability to various conditions.

3.2. Intensity Transformation Fusion

The method involves traverse all combinations of color correction (*cci*), gamma (γ), contrast (β), and brightness (α) values, applying image processing steps, and assessing the outcomes with quality metrics to empirically find the best parameters.

Let $C = (cci_1, cci_2, ...)$ be the set of color correction intensity values, $G = (\gamma_1, \gamma_2, ...)$ be the set of gamma correction values, $A = (\alpha_1, \alpha_2, ...)$ be the set of alpha values and $B = (\beta_1, \beta_2, ...)$ be the set of beta values:

$$M = \{ (cci_i, \gamma_j, \alpha_k, \beta_l) \mid cci_i \in C, \gamma_j \in G, \alpha_k \in A, \beta_l \in B \}.$$
 (1)

We provide an overview of the mentioned techniques showcasing the step-by-step, while further in-depth information can be found in the reference [Martinho et al. 2022].

This method applies a color correction through RGB histogram stretching, using a 256-value Look-up Table based on min and max values from a linear curve for histogram equalization. The transformation function f scales the image's pixel values to a new [min, max] range, where v is the original gray value, and I^{min} , I^{max} are the underwater image's extreme gray values.

$$f(v) = (v - I^{min})(max - min)/(I^{max} - I^{min}) + min$$
(2)

The second technique is the gamma correction of underwater images. In this stage, the image pixel intensities are scaled from the range [0, 255] to [0, 1.0], where I is the underwater image and g corresponds to the gamma constant value. I_G is the underwater image with corrected gamma factor, by applying the equation:

$$I_G = I^{(1/g)} \tag{3}$$

Let v be the gray value to be transformed, I an underwater image and I_{smooth} the smoothed underwater image, the Unsharp Mask filter is used to enhance edges in underwater images. In this sense, it subtracts the smoothed version of the underwater image from its original underwater image, to highlight edges through the h function. The mentioned process is performed as in the equation below:

$$h(v) = I(v) - I_{smooth}(v) \tag{4}$$

Next, the algorithm Contrast-Limited Adaptive Histogram Equalization (CLAHE) is applied. This method operates on small regions of the image, called "tiles", in which the neighborhood pixels are combined using bilinear interpolation to remove the artificial boundaries. Thus, addressing and enhancing the image contrast. This evens out gray values distribution, making hidden features more visible. The expression of modified gray levels with Uniform Distribution is given by the equation below:

$$I_{he} = [I^{max} - I^{min}] * P(I) + I^{max}$$
(5)

Let I_1 and I_2 be the processed underwater images in each layer of the proposed approach. I_1 and I_2 are combined using the linear blending as image fusion technique, represented by the k function. Thereby, the visual features in I_1 and I_2 are fused in order to improve the quality and visibility of degraded underwater images. The linear blending operation was applied as below:

$$k(v) = (1 - \alpha)I_1(v) + \alpha I_2(v)$$
(6)

The generated fused underwater image undergoes brightness and contrast adjustment, and finally yields the final output of the proposed approach. To achieve brightness and contrast adjustment transformations, each output pixel value depends only on the corresponding input pixel value, by multiplication and addition with a constant. The parameters $\beta > 0$ and γ control the contrast and brightness, respectively. The equation below presents the brightness and contrast transformation:

$$I_o = \beta * I + \gamma \tag{7}$$

4. Experiments

This section evaluates the proposed approach, conducting both qualitative and quantitative assessments. The experimental evaluation involves the comparison of our approach with traditional methods as well as recent state-of-the-art techniques based on deep learning. The experiments were conducted using Ubuntu 20.04 operating system in a Lenovo laptop with an Intel[®] CoreTM i7-10750H CPU @ 2.60GHz, 16 GB DDR4 main memory and NVIDIA[®] GeForce[®] RTX 3060 6 GB GDDR6. Furthermore, the OpenCV and Tensorflow frameworks (open source softwares) were used to support the development of the proposed approach for underwater image restoration.

4.1. Experimental Datasets

The experiments were performed using two well-stated underwater image datasets, U45 and UIEB. These datasets comprise a collection of underwater images from the ocean and are commonly used in the literature [Li et al. 2020c][Tao et al. 2020]. The U45 dataset consists of 45 images depicting natural underwater environments, while the UIEB dataset consists of 890 images capturing diverse natural underwater scenes. It is important to highlight that the U45 dataset does not present reference images, while the UIEB dataset includes reference images, enabling the comparative analysis and evaluation.

In this paper we introduce a collection of subaquatic images, called Amazon Underwater Image Dataset (AUID), acquired using a GoPro Hero 10 camera. The proposed AUID dataset consists of a challenging underwater image dataset, composed by 276 images from the Urubu river. The Urubu river is one of the tributaries of the Amazon river and is a major river in the Amazon Basin. The Urubu river plays a fundamental role in the overall hydrology and ecosystem of the Amazon region. The underwater images in the AUID dataset present high turbidity level, high scattering and low lighting in water, besides a dark color. Figure 2 presents examples of underwater images which comprise the AUID dataset.



Figure 2. Samples representing subaquatic environments in AUID dataset.

4.2. Qualitative Evaluation

In this experiment we intend to assess the effectiveness of our proposed underwater image restoration technique in comparison to other existing techniques. In this evaluation, we examine the visual quality and perceptual improvements achieved by our approach in restoring underwater images. By comparing the results of our technique with those obtained from other comparison techniques, we aim at identifying differences in terms of image clarity, visibility and overall visual appearance. In order to qualitatively evaluate the proposed approach, several existing techniques were implemented and used to restore underwater images. The U45 dataset was evaluated using the techniques: VRE [Fu et al. 2014], UDCP [Drews et al. 2016], IBLA [Peng and Cosman 2017], CBF [Ancuti et al. 2018], GDCP [Peng et al. 2018], UWCNN (UCNN) [Li et al. 2020a], WaterNet (WN) [Li et al. 2020b] and LI [Zhuang et al. 2021]. Figure 3 presents restored underwater images by our proposed approach and the comparison techniques, with respect to the U45 and UIEB dataset, enabling the visual assessment of the subaquatic images.



Figure 3. Qualitative comparison of restored underwater images on U45 (1st and 2nd rows) and UIEB (3rd and 4th rows) datasets. From left to right: raw image, VRE, UDCP, IBLA, CBF, GDCP, UCNN, WN, LI and our method.

Our approach was also qualitatively evaluated using the proposed AUID dataset. For this, were considered the techniques: Histogram Equalization (HE) [Komal and Yaduvir 2011], UDCP, IBLA and WN. Figure 4 presents restored underwater images by our proposed approach and the comparison techniques, regarding the AUID dataset, enabling the visual assessment of the subaquatic images.



Figure 4. Qualitative comparison of restored underwater images on the proposed AUID dataset. From left to right are presented the raw underwater images and the results of HE, UDCP, IBLA, WN and our method.

For U45 dataset we can see that some comparison techniques present quality results, like WN and LI. Nevertheless, aspects like high saturation, low turbidity reduction and excessive green and blue channel compensation affect the resulting underwater images. The IBLA method yields restored images highly saturated. The UCDP, IBLA, WN and LI methods does not reduce the turbidity properly in restored underwater images. Finally, the UCDP, IBLA and GDCP methods result in excessive green and blue channels compensation. Regarding UIEB dataset it is possible to observe good results obtained using the comparison techniques VRE and LI. However, in some scenarios, degraded restored images are generated, like in UCDP, IBLA, GDCP, UCNN and WN methods, with an excessive green and blue channels compensation. The GDCP, CBF, LI and UCNN methods, does not reduce the turbidity in restoring process satisfactorily. Finally, the VRE, LI and CBF methods result in restored images with high saturation. Finally, for the AUID dataset, the HE, UCDP and IBLA methods does not reduced significantly turbidity in restored images. The UCDP and WN methods yield images with low contrast, while the UCDP method generated underwater dark images. Our approach, regarding the proposed U45, UIEB and AUID dataset, achieved visually relevant results, especially due to the datasets difficulty, demonstrating the effectiveness of the proposed methodology, even in different subaquatic scenarios.

4.3. Quantitative Evaluation

In this experiment we intend to quantify the accuracy of the proposed approach for underwater image restoration. For this evaluation we have used measurements and metrics to quantitatively assess and compare the obtained results. For this experiment, have been used the full reference image quality metrics: Entropy, PSNR and SSIM. More results regarding non-reference image quality metrics can be found in [Martinho et al. 2024].

In the quantitative analysis, regarding the U45 and UIEB datasets, presented in the Table 1, are compared the proposed approach and the algorithms VRE, UDCP, IBLA,

Method	Entropy †		PSNR ↑		SSIM ↑	
	U45	UIEB	U45	UIEB	U45	UIEB
Raw	6.144	6.939	17.215	19.856	0.538	0.633
VRE	6.597	7.627	22.405	21.277	0.786	0.654
UDCP	5.513	7.109	21.838	23.673	0.684	0.675
IBLA	6.963	7.348	25.674	20.142	0.567	0.653
CBF	5.458	7.423	22.424	26.785	0.794	0.740
GDCP	6.165	7.343	23.601	25.254	0.758	0.788
UWCNN	6.032	6.592	26.879	23.458	0.585	0.677
WN	7.278	7.166	26.386	25.157	0.554	0.785
LI	7.924	7.718	26.714	27.157	0.831	0.756
Ours	7.936	7.784	26.967	27.299	0.847	0.793

 Table 1. Full reference image quality assessment regarding Entropy, PSNR and SSIM on U45 and UIEB image datasets

Table 2. Full reference image quality assessment regarding Entropy, PSNR and SSIM on AUID dataset images

Method	Entropy ↑	PSNR ↑	SSIM ↑
Raw	4.037	17.522	0.300
HE	4.422	26.048	0.586
UDCP	4.787	25.897	0.411
IBLA	6.475	26.085	0.621
WN	7.100	27.087	0.662
Ours	7.1975	27.553	0.667

CBF, GDCP, UWCNN, WN and LI, using full reference image quality metrics. For the Entropy, PSNR and SSIM quality metrics the WN and LI restoration techniques presented the best results among the comparison techniques. Meanwhile, regarding the AUID dataset, are compared the proposed approach and the algorithms HE, UDCP, IBLA and WN. From the Table 2, it was possible to verify that for the Entropy, PSNR and SSIM metrics the WN restoration technique presented the best results among the comparison techniques. It is important to highlight that our approach overcome all the comparison techniques, even in very challenging datasets, with high turbidity and low visibility.

5. Conclusion and Future Work

This paper presented an approach to underwater image quality enhancement using deep learning and intensity transformation techniques. A CNN was used for learning the best parameters needed for the image transformation functions, such as contrast adjustment, histogram equalization and gamma correction. By combining these transformations using the parameters found by the CNN, our approach achieved robust results as presented by the experiments. In this work we have created a new underwater image dataset, namely Amazon Underwater Image Dataset (AUID). The AUID dataset is composed by 276 images from the Urubu river, which is a blackwater tributary of the Amazon river. This dataset shows the potential for the acquisition and use of underwater images from the Amazon region, particularly the blackwater rivers. As future directions of this work we intend to investigate different approaches for underwater image restoration, like using High Dynamic Range (HDR) in subaquatic scenarios. We also intend to expand the sets of underwater images in experimental process, in order to propose an invariant to water condition underwater image restoration technique, addressing the most of the subaquatic scenarios. We also intend to improve and significantly increase the number of images in our underwater dataset (AUID).

References

- Ancuti, C. O., Ancuti, C., De Vleeschouwer, C., and Bekaert, P. (2018). Color balance and fusion for underwater image enhancement. *IEEE Trans. on Image Processing*, 27(1):379–393.
- Drews, P. L., Nascimento, E. R., Botelho, S. S., and Montenegro Campos, M. F. (2016). Underwater depth estimation and image restoration based on single images. *IEEE Computer Graphics and Applications*, 36(2):24–35.
- Fu, X., Zhuang, P., Huang, Y., Liao, Y., Zhang, X.-P., and Ding, X. (2014). A retinexbased enhancing approach for single underwater image. In 2014 IEEE International Conference on Image Processing (ICIP), pages 4572–4576.
- Gangisetty, S. and Rai, R. R. (2022). Floodnet: Underwater image restoration based on residual dense learning. *Signal Processing: Image Communication*, 104:116647.
- Huang, Y., Yuan, F., Xiao, F., and Cheng, E. (2022). Underwater image enhancement based on color restoration and dual image wavelet fusion. *Signal Processing: Image Communication*, 107:116797.
- Jiang, Qiuping Li, F. L. C. L. D. D. J. A. (2022). Underwater imaging formation modelembedded multiscale deep neural network for underwater image enhancement. *Mathematical Problems in Engineering*, 13(4).
- Kai, Y., Lanyue, L., Ziqiang, Z., Guoqing, W., and Yang, Y. (2022). Medium transmission map matters for learning to restore real-world underwater images.
- Komal, V. and Yaduvir, S. (2011). Enhancement of images using histogram processing techniques. *International Journal of Computer Technology and Applications*, 02.
- Li, C., Anwar, S., and Porikli, F. (2020a). Underwater scene prior inspired deep underwater image and video enhancement. *Pattern Recognition*, 98:107038.
- Li, C., Guo, C., Ren, W., Cong, R., Hou, J., Kwong, S., and Tao, D. (2020b). An underwater image enhancement benchmark dataset and beyond. *IEEE Transactions on Image Processing*, 29:4376–4389.
- Li, C., Tang, S., Kwan, H. K., Yan, J., and Zhou, T. (2020c). Color correction based on cfa and enhancement based on retinex with dense pixels for underwater images. *IEEE Access*, 8:155732–155741.
- Liu, S., Fan, H., Lin, S., Wang, Q., Ding, N., and Tang, Y. (2022a). Adaptive learning attention network for underwater image enhancement. *IEEE Robotics and Automation Letters*, 7(2):5326–5333.

- Liu, Y., Xu, H., Zhang, B., Sun, K., Yang, J., Li, B., Li, C., and Quan, X. (2022b). Model-based underwater image simulation and learning-based underwater image enhancement method. *Information*, 13(4).
- Lyu, Z., Peng, A., Wang, Q., and Ding, D. (2022). An efficient learning-based method for underwater image enhancement. *Displays*, 74:102174.
- Martinho, L., Neto, O., Calvalcanti, J., Pio, J., and Oliveira, F. (2023). An approach for fish detection in underwater images. In *Anais do XVIII Workshop de Visão Computacional*, pages 6–11, Porto Alegre, RS, Brasil. SBC.
- Martinho, L. A., Calvalcanti, J. a. M. B., Pio, J. L., and Oliveira, F. G. (2024). Diving into clarity: Restoring underwater images using deep learning. *J. Intell. Robotics Syst.*, 110(1).
- Martinho, L. A., Oliveira, F. G., Cavalcanti, J. M. B., and Pio, J. L. S. (2022). Underwater image enhancement based on fusion of intensity transformation techniques. In 2022 *Latin American Robotics Symposium (LARS)*, 2022 Brazilian Symposium on Robotics (SBR), and 2022 Workshop on Robotics in Education (WRE), pages 348–353.
- Peng, Y.-T., Cao, K., and Cosman, P. C. (2018). Generalization of the dark channel prior for single image restoration. *IEEE Trans. on Image Processing*, 27(6):2856–2868.
- Peng, Y.-T. and Cosman, P. C. (2017). Underwater image restoration based on image blurriness and light absorption. *IEEE Trans. on Image Processing*, 26(4):1579–1594.
- Qiao, N., Dong, L., and Sun, C. (2023). Adaptive deep learning network with multi-scale and multi-dimensional features for underwater image enhancement. *IEEE Transactions on Broadcasting*, 69(2):482–494.
- Sharma, P. K., Bisht, I., and Sur, A. (2022). Wavelength-based attributed deep neural network for underwater image restoration.
- Tao, Y., Dong, L., and Xu, W. (2020). A novel two-step strategy based on white-balancing and fusion for underwater image enhancement. *IEEE Access*, 8:217651–217670.
- Wang, Y., Zhao, Y., Pan, H., and Zhou, W. (2022). An improved reinforcement learning method for underwater image enhancement. In 2022 IEEE 25th Int. Conf. on Computer Supported Cooperative Work in Design (CSCWD), pages 1077–1082.
- Zhao, W., Rong, S., Li, T., Feng, J., and He, B. (2023). Enhancing underwater imagery via latent low-rank decomposition and image fusion. *IEEE Journal of Oceanic Engineering*, 48(1):147–159.
- Zheng, M. and Luo, W. (2022). Underwater image enhancement using improved cnn based defogging. *Electronics*, 11(1).
- Zhuang, P., Li, C., and Wu, J. (2021). Bayesian retinex underwater image enhancement. *Engineering Applications of Artificial Intelligence*, 101:104171.
- Zhuang, P., Wu, J., Porikli, F., and Li, C. (2022). Underwater image enhancement with hyper-laplacian reflectance priors. *IEEE Trans. on Image Processing*, 31:5442–5455.